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Review Questions for Test #2

MATHEMATICS 201-009
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TEST #2

1. Find all the zeros of the function. Do not sketch its graph.

$$f(x) = 2x^3 - 7x^2 + 4x + 4$$

Solution $\boxed{\frac{p}{q}\text{-method.}}$

p: factor of $a_0 = 4$
q: factor of $a_n = 2$

- $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q}$: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2}$

Possibilities for $\frac{p}{q}$: $1, -1, 2, -2, \frac{1}{2}, -\frac{1}{2}, 4, -4$

- Use ~~Synth~~ Synthetic-division to try to find zeros

$$\begin{array}{r|rrrr} 1 & 2 & -7 & 4 & 4 \\ & & 2 & -5 & -1 \\ \hline & 2 & -5 & -1 & 3 \end{array}$$

$x = 1$ is NOT a zero

$$\begin{array}{r|rrrr} -1 & 2 & -7 & 4 & 4 \\ & & -2 & 9 & -13 \\ \hline & 2 & -9 & 13 & -9 \end{array}$$

$x = -1$ is not a zero

$$\begin{array}{r|rrrr} 2 & 2 & -7 & 4 & 4 \\ & & 4 & -6 & -4 \\ \hline & 2 & -3 & -2 & 0 \end{array}$$

$x = 2$ is a zero

Then $f(x)$ is factored as

$$2x^3 - 7x^2 + 4x + 4 = (x-2)(2x^2 - 3x - 2)$$
$$= (x-2)(2x+1)(x-2)$$

So, all zeros are:

$$x_1 = -\frac{1}{2}, \quad x_2 = x_3 = 2 \qquad //$$

2. Find the domain, the x and y intercepts, symmetry, the asymptotes and an appropriate table of values, then sketch the graph of $y = f(x) = \dfrac{2x^2}{x^2 - 1}$.

Solution: • Domain: $D = \{x \mid x^2 - 1 \neq 0\}$.

$$x^2 - 1 \neq 0 \Rightarrow (x-1)(x+1) \neq 0$$
$$\Rightarrow x \neq 1 \ \& \ x \neq -1$$
$$D = \{x \mid x \neq 1 \ \& \ x \neq -1\}$$
$$= (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$$

• x-intercept:

put $y = 0 \Rightarrow \dfrac{2x^2}{x^2-1} = 0 \Rightarrow x = 0$.

the x-intercept is $(0, 0)$

• y-intercept:

put $x = 0$. then $y = f(0) = 0$

the y-intercept is (0.0).

• Symmetry:

Since $f(-x) = \dfrac{2(-x)^2}{(-x)^2 - 1} = \dfrac{2x^2}{x^2-1} = f(x)$

$f(x)$ is even, which is symmetric on the y-axis (reflection accross the y-axis)

• asymptotes:

vertical asymptotes: $x^2 - 1 = 0$

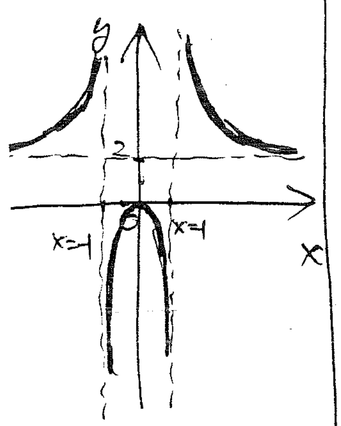
$$\Rightarrow x = \pm 1$$

horizontal asymptotes:

$$\cancel{\lim_{x \to \pm\infty}} \frac{2x^2}{x^2-1} = \frac{2x^2/x^2}{(x^2-1)/x^2} = \frac{2}{1 - \frac{1}{x^2}}$$
$$\longrightarrow \frac{2}{1-0} = 2 \quad \text{as } x \to \pm\infty.$$

So, $y = 2$ is the horizontal asymptote

• Graph:



x	y
-3	$\frac{9}{4}$
$\frac{1}{2}$	$-2/3$

3. Given $f(x) = \sqrt{x-1}$ and $g(x) = x^2 - 1$, find the following new functions and their domains.

(a) $(f + 2g)(x) = \boxed{\sqrt{x-1} + 2(x^2-1)}$

Domain : $\cancel{(-\infty, \infty)}$ $x-1 \geq 0 \Rightarrow \boxed{x \geq 1}$

(b) $\left(\frac{g}{f}\right)(x) = \boxed{\dfrac{x^2-1}{\sqrt{x-1}}}$

Domain: $\sqrt{x-1} \neq 0 \Rightarrow \begin{cases} x-1 \geq 0 \\ x-1 \neq 0 \end{cases} \Rightarrow x-1>0$

$\Rightarrow \boxed{x>1}$

c) $(f \cdot g)(x) = \boxed{\sqrt{x-1}\,(x^2-1)}$

D: $x-1 \geq 0 \Rightarrow \boxed{x \geq 1}$

(d) $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)-1} = \sqrt{(x^2-1)-1} = \boxed{\sqrt{x^2-2}}$

D: $x^2-2 \geq 0 \Rightarrow (x-\sqrt{2})(x+\sqrt{2}) \geq 0$

$\Rightarrow \boxed{x \geq \sqrt{2}}$ or $\boxed{x \leq -\sqrt{2}}$

(e) $(g \circ g)(x) = g(g(x)) = (g(x))^2 - 1 = \boxed{(x^2-1)^2 - 1}$

D: $\boxed{(-\infty, \infty)}$

4. Find the vertex, the x and y intercepts, then sketch the graph of $f(x) = 2x^2 - 2x - 4$.

$$f(x) = 2(x^2 - x - 2) = 2x^2 - 2x - 4, \quad a = 2,\ b = -2,\ c = -4$$

- Vertex: $h = -\frac{b}{2a} = -\frac{-2}{2} = 1$

$$k = \frac{4ac - b^2}{4a} = \frac{4 \cdot 2 \cdot (-4) - (-2)^2}{4 \cdot 2} = -\frac{9}{2}$$

Vertex is $(1, -\frac{9}{2})$

- x-intercept: $f(x) = 2(x^2 - x - 2) = 2(x+1)(x-2) = 0$

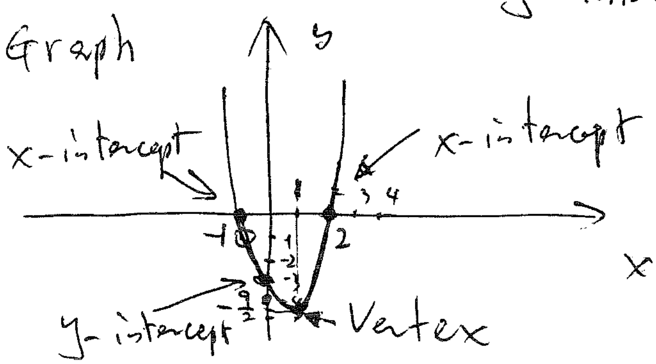
$\Rightarrow x = -1, \quad x = 2.$

So the x-intercept: $(-1, 0), \ (2, 0)$

- y-intercept: put $x = 0$: $y = f(0) = -4$

So, the y-intercept is $(0, -4)$

- Graph



5. Solve the inequality $-4 - \frac{1}{x} \geq -\frac{2}{x}$.

$$-4 - \frac{1}{x} + \frac{2}{x} \geq 0$$

$$-\frac{4x}{x} - \frac{1}{x} + \frac{2}{x} \geq 0$$

$$\frac{-4x - 1 + 2}{x} \geq 0$$

$$\frac{-4x + 1}{x} \geq 0$$

Critical Value:

$x = 0$ (zero ~~of~~ denominator)

$x = \frac{1}{4}$ (zero of numerator)

The divided intervals are:

$(-\infty, 0), \ (0, \frac{1}{4}], \ [\frac{1}{4}, \infty)$

$-\infty$ ⊖ 0 ⊕ $\frac{1}{4}$ ⊖ ∞

choose: $x = -1$, test: $\frac{-4(-1)+1}{-1} = -5 \ (<0)$

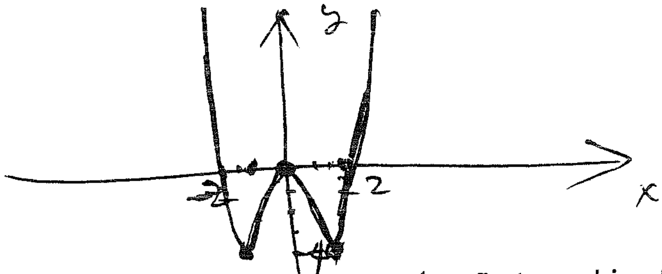
choose: $x = \frac{1}{8}$: test: $\frac{-4 \cdot \frac{1}{8} + 1}{\frac{1}{8}} = 4 \ (>0)$

choose: $x = 1$: test: $\frac{-4+1}{1} = -3 \ (<0)$

So, the solution is: $\boxed{0 < x \leq 1/4.}$

6. Find if f(x) is even or odd. Find its intercepts, then find appropriate table of values to sketch the graph of $f(x) = x^4 - 4x^2$.

- $f(x) = x^4 - 4x^2$ is even, because:
  $f(-x) = (-x)^4 - 4(-x)^2 = x^4 - 4x^2 = f(x)$.
- x-intercepts: $0 = x^4 - 4x^2 = x^2(x^2-4) = x^2(x-2)(x+2)$
  $\Rightarrow x_1 = 0,\ x_2 = 2,\ x_3 = -2$.
- y-intercept: $y = f(0) = 0$.
- Graph

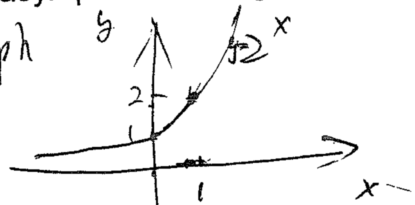


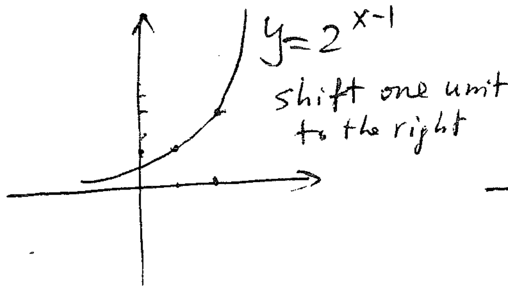
7. Use shifts and reflections to graph $f(x) = y = 3 - 2^{x-1}$ by first graphing the following functions:

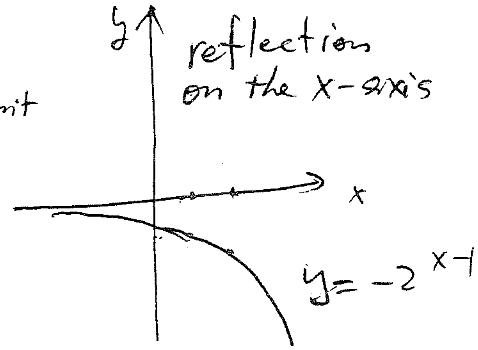
$$y = 2^x,\quad y = 2^{x-1},\quad y = -2^{x-1}.$$

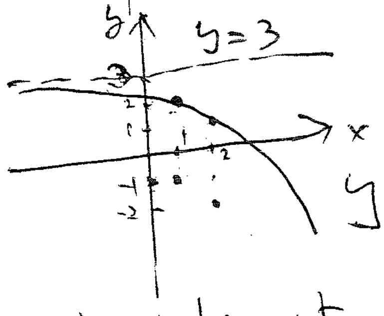
Also find the x and y intercepts and the equation of the asymptote of f(x). Show the asymptote on the graph.

- Graph









shift 3 units upward.

$y = 3 - 2^{x-1}$

- x-intercept: $0 = 3 - 2^{x-1} \Rightarrow 2^{x-1} = 3$
  $\log_2 2^{x-1} = \log_2 3 \Rightarrow (x-1)\log_2 2 = \log_2 3$
  $\Rightarrow x - 1 = \log_2 3 \Rightarrow \boxed{x = \log_2 3 + 1}$

- y-intercept:

  - $y = 3 - 2^{0-1} = 3 - 2^{-1} = 3 - \frac{1}{2} = \boxed{\frac{5}{2}}$

- Asymptote: $\boxed{y = 3}$

8. Given $f(x) = x^3 - 1$, find:

(a) $f^{-1}(x)$, the inverse function of $f(x)$.

(b) the domain and the range for both $f(x)$ and $f^{-1}(x)$.

(c) Graph $f(x)$ and $f^{-1}(x)$ on the same co-ordinate system.

(d) Find $(f \circ f^{-1})(x)$.

(e) Find the axis of symmetry between $f(x)$ and $f^{-1}(x)$.

(a). $y = x^3 - 1$ its inverse: $x = y^3 - 1$

$\Rightarrow y^3 = x + 1 \Rightarrow \boxed{y = \sqrt[3]{x+1}}$

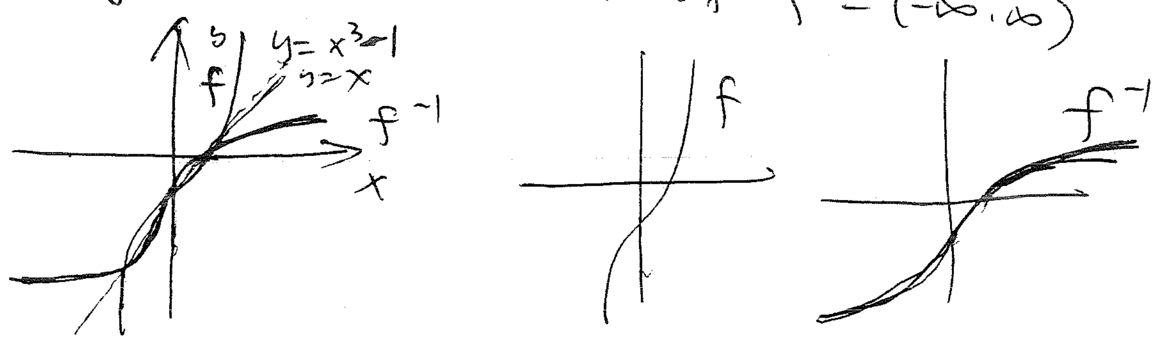
(b) Domain of f: $D = (-\infty, \infty)$

Range of f: $R = (-\infty, \infty)$

Domain of f^{-1} = Range of $f = (-\infty, \infty)$

~~Range~~ Range of f^{-1} = Domain of $f = (-\infty, \infty)$

(c)



(d) $(f \circ f^{-1})(x) = x$.

(e) $\boxed{y = x}$

9. Solve the equation $\log_2 x + \log_2(x-2) = 3$.

$$\log_2 x + \log_2 (x-2) = 3$$

$$\log_2 [x(x-2)] = 3\cdot 1$$

$$\log_2 (x(x-2)) = 3\cdot \log_2 2$$

$$\log_2 (x(x-2)) = \log_2 2^3$$

$$x(x-2) = 2^3$$

$$x^2 - 2x - 8 = 0$$

$$x_1 = \frac{-(-2) + \sqrt{(-2)^2 - 4\cdot 1\cdot(-8)}}{2} = \frac{2+\sqrt{36}}{2} = 4$$

$$x_2 = \frac{-(-2) - \sqrt{36}}{2} = -2$$

Since we need.
$x > 0$ for $\log_2 x$
$x-2>0$ for $\log_2(x-2)$
The true solution is.
$\boxed{x=4}$.

10. Given $\log_a 2 = 0.3$, $\log_a 5 = 0.7$, $\log_a 3 = 0.4$, find $\log_a\left(\frac{\sqrt[4]{50}}{30}\right)$.

$$\log_a\left(\frac{\sqrt[4]{50}}{30}\right) = \log_a \sqrt[4]{50} - \log_a 30$$

$$= \log_a (50)^{\frac{1}{4}} - \log_a (2\cdot 3\cdot 5)$$

$$= \frac{1}{4}\log_a (2\cdot 5\cdot 5) - \log_a (2\cdot 3\cdot 5)$$

$$= \frac{1}{4}(\log_a 2 + \log_a 5 + \log_a 5) - (\log_a 2 + \log_a 3 + \log_a 5)$$

$$= \frac{1}{4}(0.3 + 0.7 + 0.7) - (0.3 + 0.4 + 0.7)$$

$$= \frac{1}{4}\cdot 1.7 - 1.4 = \boxed{-0.975.}$$

11. Solve the inequality $\left|\frac{x-3}{4}\right| > +10$.

$$\frac{x-3}{4} > 10 \quad \text{or} \quad \frac{x-3}{4} < -10$$

$$x-3 > 40 \quad \text{or} \quad x-3 < -40$$

$$\boxed{x > 43} \quad \text{or} \quad \boxed{x < -37}$$