Review Questions for Final Exam
MATH 201-009

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1. Consider the following:

   $0, \ -14, \ 23.77, \ 5, \ \sqrt{2}, \ \frac{0}{1}, \ \frac{\sqrt{2}}{\sqrt{2}}, \ 0.23\overline{023}, \ e$

   (a) which are integers ?
   (b) which are rational numbers ?
   (c) which are irrational numbers ?
   (d) which are real numbers ?
   (e) which are natural numbers ?

2. Simplify $\dfrac{\frac{x}{y} - \frac{y}{x}}{\frac{x-y}{xy}}$

3. Given $f(x) = x^2 + x$ and $g(x) = -3x + 7$, find:

   (a) $(f+g)(x)$; (b) $(f \cdot g)(x)$; (c) $\left(\frac{f}{g}\right)(x)$;

   (d) $(f \circ g)(x)$; (e) $(g \circ f)(x)$.

4. Find an equation of the straight line that passes through the point $A(-2, 4)$ and is perpendicular to the line $x - 2y + 4 = 0$

5. Given the function $f(x) = \frac{x^2}{x^2-4}$

(a) Find the domain and the range of $f(x)$.

(b) Find the x and y intercepts

(c) Check if $f(x)$ is even or odd or neither

(d) Find the vertical asymptotes, the horizontal asymptotes, if possible.

(e) Sketch the graph of $f(x)$.

6. Graph $f(x) = e^x$ first, then $g(x) = 3 - e^{-x}$.

7. Solve the equation: $\log_3(x+1) + \log_3(x+3) = 1$

8. Prove: $\frac{\sin(\alpha+\beta)}{\cos\alpha \cdot \cos\beta} = \tan\alpha + \tan\beta$

9. Solve: $2\cos 2x - 1 = 0$ for $x \in [0°, 360°)$

10. Let $\triangle ABC$ be $A = 30°$, $B = 45°$, $c = 10$.
Solve this triangle.

Solutions to Review Questions

1. (a) integers: $0, -14, 5, \frac{\sqrt{2}}{\sqrt{2}}, \frac{0}{1}$ $=1$ $=0$

(b) rational numbers: $0, -14, 23.77, 5, \frac{0}{1}, \frac{\sqrt{2}}{\sqrt{2}}$
$0.23\overline{023}$

(c) irrational numbers: $\sqrt{2}, e$

(d) real numbers: all of them

(e) natural numbers: $5, \frac{\sqrt{2}}{\sqrt{2}} = 1$

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2. $$\frac{\frac{x}{y} - \frac{y}{x}}{\frac{x-y}{xy}} = \frac{\frac{x^2 - y^2}{yx}}{\frac{x-y}{xy}}$$

$$= \frac{x^2 - y^2}{yx} \div \frac{x-y}{xy}$$

$$= \frac{(\cancel{x-y})(x+y)}{\cancel{xy}} \times \frac{\cancel{xy}}{\cancel{x-y}}$$

$$= x + y$$

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3. (a) $(f+g)(x) = f(x) + g(x)$
$= (x^2 + x) + (-3x + 7)$
$= x^2 - 2x + 7$

(b) $(f \cdot g)(x) = (f(x)) \cdot (g(x))$

$= (x^2+x)\cdot(-3x+7)$

$= -3x^3 + 4x^2 + 7x$

(c) $\left(\frac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2+x}{-3x+7}$

(d) $(f \circ g)(x) = f(g(x)) = [g(x)]^2 + [g(x)]$

$= (-3x+7)^2 + (-3x+7)$

$= 9x^2 - 42x + 49 - 3x + 7$

$= 9x^2 - 45x + 56$

(e) $(g \circ f)(x) = -3[f(x)] + 7$

$= -3\cdot(x^2+x) + 7$

$= -3x^2 - 3x + 7$

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4. Let the equation of the line be:

$l_1: \quad y = m_1 x + b_1$

For the second line: $x - 2y + 4 = 0$

i.e. $2y = x + 4 \Rightarrow y = \frac{1}{2}x + 2$

its slope $m_2 = \frac{1}{2}$.

Note that the first line l_1 is

perpendicular to the second line l_2, we then have

$$m_1 \cdot m_2 = -1$$

i.e. $m_1 = -\frac{1}{m_2} = -\frac{1}{\frac{1}{2}} = -2$.

So. l_1: $y = -2x + b_1$.

In order to solve for b_1, we know the first line passes through the point $A(-2,4)$, then we must have

$$4 = -2 \cdot (-2) + b_1$$

which gives $b_1 = 0$

Thus, the line l_1 is:

$$l_1: \boxed{y = -2x}$$

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5. (a) domain: $x^2 - 4 \neq 0$. $(x-2)(x+2) \neq 0$

$x \neq -2$, $x \neq 2$.

$$D = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$$

Range: $R =$

(b) x-intercepts:

$$\frac{x^2}{x^2-4}=0 \Rightarrow x^2=0 \Rightarrow \boxed{x=0}$$

y-intercepts:

$$y=\frac{x^2}{x^2-4}\Bigg|_{x=0}=\frac{0^2}{0^2-4}=0$$

i.e. $\boxed{y=0}$

(c) $f(x)=\dfrac{x^2}{x^2-4}$,

$$f(-x)=\frac{(-x)^2}{(-x)^2-4}=\frac{x^2}{x^2-4}=f(x)$$

So $f(x)$ is even.

(d) Vertical asymptotes:

(the points such that the denominator is 0.)

$$x^2-4=0 \Rightarrow \boxed{x=-2},\ \boxed{x=2}$$

horizontal asymptotes:

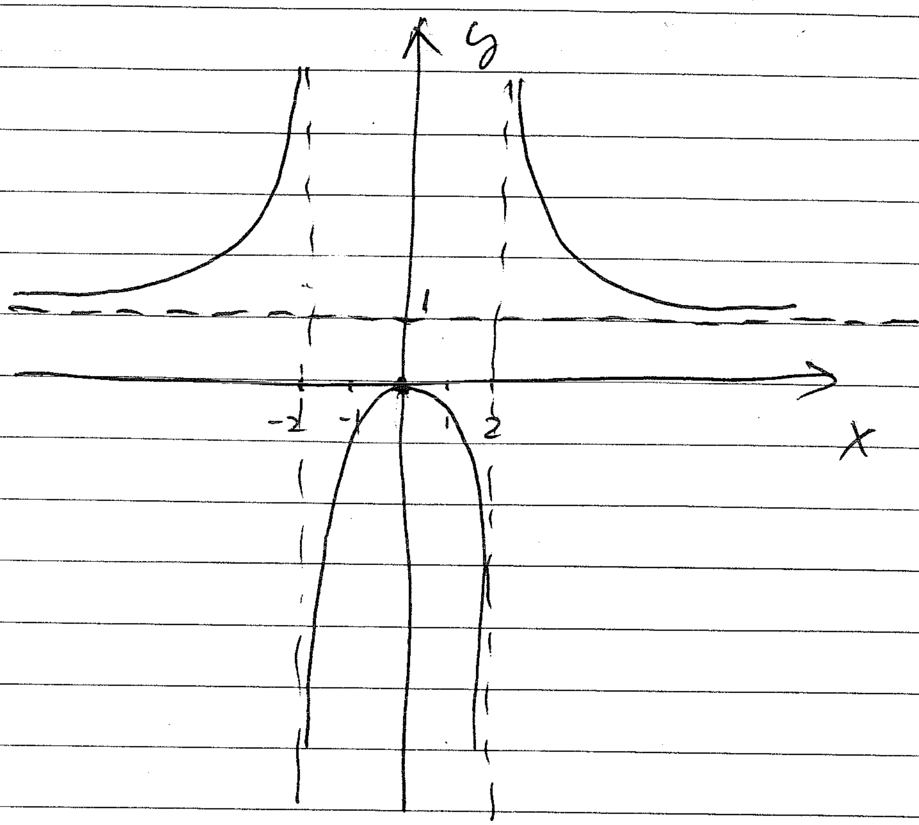
$$y=\frac{x^2}{x^2-4}=\frac{x^2/x^2}{(x^2-4)/x^2}$$

$$=\frac{1}{\frac{x^2}{x^2}-\frac{4}{x^2}}=\frac{1}{1-\frac{4}{x^2}}\rightarrow\frac{1}{1-0}=1$$

as $x\to\pm\infty$.

$\boxed{y=1}$ is the horizontal asymptote.

(e)

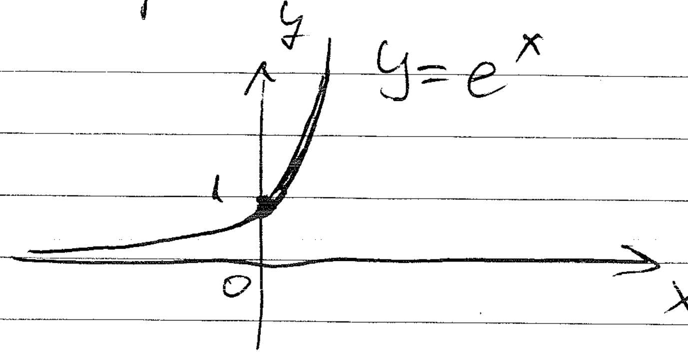


Test points:

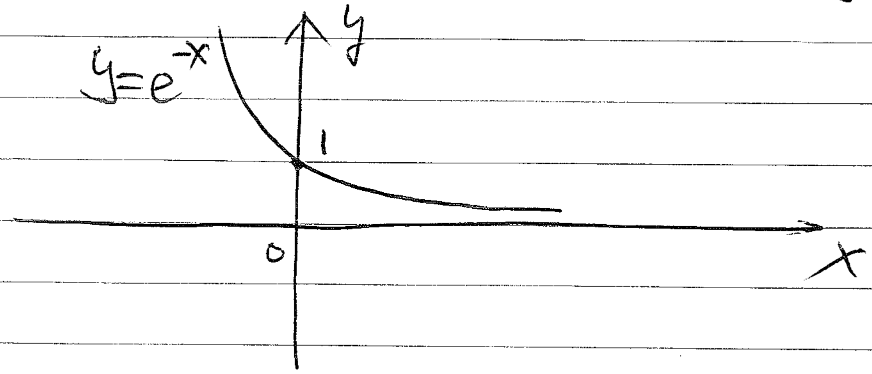
	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
$f(x)$	$x=-3$ $f(-3)=\frac{9}{9-4}=\frac{9}{5}$ $+$	$x=-1$ $f(-1)=\frac{(-1)^2}{(-1)^2-4}$ $=\frac{1}{-3}$ $-$	$x=3$ $f(3)=\frac{9}{5}$ $+$

• Graph of $y = e^x$:

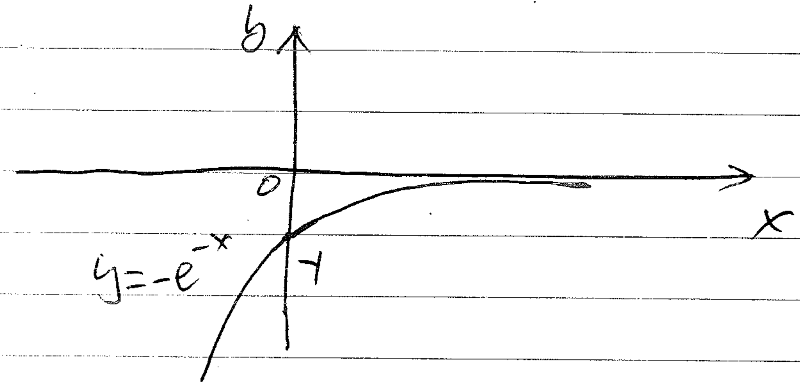
6.



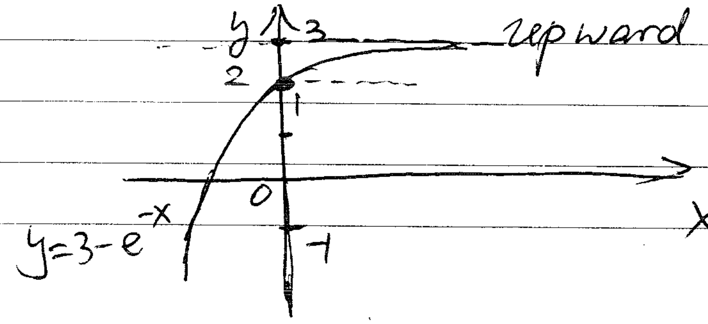
• $y = e^{-x}$ is a reflection of $y = e^x$ accross the y-axis



• $y = -e^{-x}$ is a reflection of $y = e^{-x}$ accross the x-axis



• $y = 3 - e^{-x}$ is by shifting $y = -e^{-x}$ upward 3 units.



7. $\log_3(x+1) + \log_3(x+3) = 1$

$\log_3 (x+1)\cdot(x+3) = 1 = \log_3 3$

$(x+1)(x+3) = 3$

$x^2 + 4x + 3 = 3$

$x^2 + 4x = 0$

$x(x+4) = 0$

$x_1 = 0, \quad x_2 = -4$

For $\log_3(x+1)$ and $\log_3(x+3)$,
by the definition of the logarithm, we need:

$x+1 > 0$ and $x+3 > 0$

i.e. $x > -1$ and $x > -3$

So, $x_1 = 0$ satisfies the above conditions, which is a true solution.
but $x_2 = -4$ doesn't satisfy the above conditions, so, it is false.
Thus, the solution is:

$$\boxed{x = 0}$$

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8. Proof.

$$LHS = \frac{\sin(\alpha+\beta)}{\cos\alpha \cdot \cos\beta} = \frac{\sin\alpha\cos\beta + \sin\beta\cos\alpha}{\cos\alpha\cos\beta}.$$

$$= \frac{\sin\alpha \cancel{\cos\beta}}{\cos\alpha \cancel{\cos\beta}} + \frac{\sin\beta \cancel{\cos\alpha}}{\cancel{\cos\alpha}\cos\beta}$$

$$= \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}$$

$$= \tan\alpha + \tan\beta$$

$$= RHS.$$

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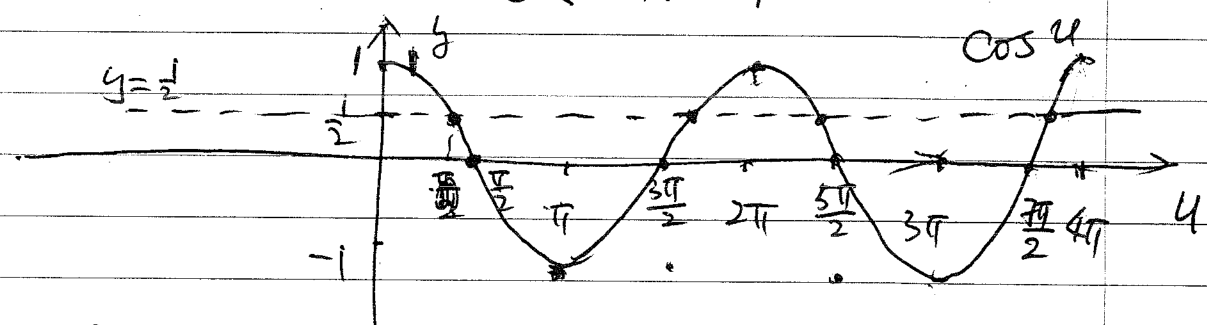
9. $2\cos 2x - 1 = 0$ for $x \in [0°, 360°)$

$2\cos 2x = 1$, $0° \le x < 360°$

$\cos 2x = \frac{1}{2}$, $0° \le 2x < 720°$

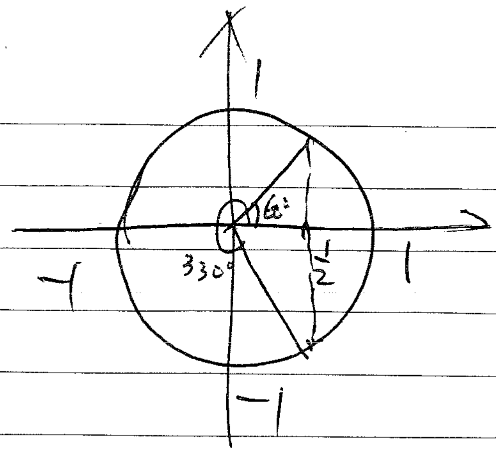


So, for $2x$ in $[0°, 720°)$ satisfying $\cos 2x = \frac{1}{2}$,

We have: $(\because \cos 60° = \frac{1}{2})$

$2x = 60°,\ 300°,\ 420°,\ 660°$

$(360°-60°)$ $(360°+60°)$ $(720°-60°)$



So, the solutions are:

$x_1 = 30°, \quad x_2 = 150°, \quad x_3 = 210°, \quad x_4 = 330°$

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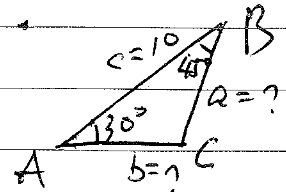
10.



$A = 30°. \quad B = 45°$

$C = 180° - A - B$

$= 180° - 30° - 45°$

$= 105°$

$a = ? \quad b = ? \quad c = 10$

By using the law of sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

we have

$$a = \frac{c}{\sin C} \sin A = \frac{10}{\sin 105°} \sin 30°$$

$$= \frac{10}{\frac{\sqrt{2}+\sqrt{6}}{2}} \cdot \frac{1}{2} = \frac{10}{\sqrt{2}+\sqrt{6}}$$

$$= \frac{10}{(\sqrt{6}+\sqrt{2})} \cdot \frac{(\sqrt{6}-\sqrt{2})}{(\sqrt{6}-\sqrt{2})} = \frac{10(\sqrt{6}-\sqrt{2})}{6-2} = \frac{5}{2}(\sqrt{6}-\sqrt{2})$$

$$b = \frac{c}{\sin C} \cdot \sin B = \frac{10}{\sin 105^\circ} \sin 45^\circ$$

$$= \frac{10}{\frac{\sqrt{2}+\sqrt{6}}{2}} \cdot \frac{\sqrt{2}}{2} = \frac{10}{1+\sqrt{3}} = \frac{10}{1+\sqrt{3}} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{10(\sqrt{3}-1)}{(\sqrt{3})^2-1^2} = \frac{10(\sqrt{3}-1)}{3-1} = 5(\sqrt{3}-1)$$