Final Review #2

# CHAMPLAIN COLLEGE ST.-LAMBERT

## MATH 201-NYB: Calculus II

## **Review Questions**

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1. Sketch the graph of the function

$$f(x) = \begin{cases} 1+x, & \text{if } \; -3 \le x \le 0 \\ 1-\sqrt{1-x^2}, & \text{if } \; 0 < x \le 1 \\ 1, & \text{if } \; 1 < x \le 3, \end{cases}$$

then evaluate the definite integral $\int_{-3}^{3} f(x)dx$ by interpreting it in terms of area (do not antidifferentiate).

2. Find the derivative of the function

$$F(x) = \int_0^{\tan(2x)} \frac{e^t\sqrt{1+t^2}}{1+t}dt.$$

3. Find the indefinite integrals:

$$\text{(a)} \int e^{\sqrt{x}}dx, \qquad \text{(b)} \int (x^2+1)\cos x dx, \qquad \text{(c)} \int \frac{(x+2)^2}{x^2+4}dx.$$

4. Evaluate the definite integrals $\int \ln^2 x dx$.

$$\text{(a)} \int_1^{e^2} \frac{\ln^3 x}{x}dx, \qquad \text{(b)} \int_0^{\frac{1}{2}} \arctan(2x)dx, \qquad \text{(c)} \int_0^4 \frac{x}{\sqrt{x^2+9}}dx.$$

5. (a) Find the area bounded by the curves $x + y^2 = 2$ and $x + y = 0$.
   (b) Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $y = 1$ about the $x$-axis.
   (c) Find the average value of the function $f(x) = \cos^2 x$ on the interval $[\frac{\pi}{2}, \pi]$.

6. Evaluate the given improper integrals or show them to be divergent:

$$\text{(a)} \int_1^{\infty} \frac{x^2+2}{x^3}dx, \qquad \text{(b)} \int_0^2 \frac{x-1}{2x-x^2}dx.$$

7. Show the convergence or divergence of the following sequences:

$$\text{(a)} \left\{\frac{(-1)^n n}{n+1}\right\}, \qquad \text{(b)} \left\{\frac{2\sqrt{n}-5}{2-5\sqrt{n}}\right\}, \qquad \text{(c)} \left\{\frac{n^3-1}{1000n^2+100000n}\right\}.$$

8. Show the convergence or divergence of the following series:

$$\text{(a)} \sum_{n=1}^{\infty} \frac{3^{1+n}}{n!}, \qquad \text{(b)} \sum_{n=1}^{\infty} \frac{n}{n^4+1}, \qquad \text{(c)} \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2n-1}}.$$

9. (a) Find the sum of the series $\sum_{n=1}^{\infty}(-2)^{-n}$.
   (b) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2 2^n}$.
   (c) Find the MacLaurin series for the function $f(x) = \frac{\sin x}{x}$.

# Answers and Hints

**Q1.** *Answer*: $\frac{3}{2} - \frac{\pi}{4}$. *Hints*: $y = 1 + x$ is a segment line on $[-3, 0]$; $y = 1 - \sqrt{1 - x^2}$ is an arc of the circle centred at (0,1) with radius 1.

**Q2.** *Answer*: $\frac{2e^{\tan(2x)}\sec^2(2x)}{\cos(2x)+\sin(2x)}$.

**Q3.** (a) *Answer*: $2e^{\sqrt{x}}\sqrt{x} - 2e^{\sqrt{x}} + C$. *Hints*: substituting $u = \sqrt{x}$ first, then integrating it by parts.

(b) *Answer*: $(x^2 + 1)\sin x + 2x\cos x - 2\sin x + C$. *Hints*: using integration by parts twice.

(c) *Answer*: $x + 2\ln(x^2 + 4) + C$.

**Q4.** (a) *Answer*: 4. *Hints*: substituting $u = \ln x$.

(b) *Answer*: $\frac{\pi}{8} - \frac{1}{4}\ln 2$. *Hints*: integration by parts, then substitution.

(c) *Answer*: 2. *Hints*: substitution $u = x^2 + 9$ Note: no trigonometric substitution is required here. If the question is changed into $\int_0^4 \frac{1}{\sqrt{x^2+4}}dx$, we need a trigonometric substitution $x = 2\tan\theta$.

**Q5.** *Answers*: (a) $A = \frac{9}{2}$. (b) $V = \frac{8}{5}\pi$. (c) $f_{average} = \frac{1}{2}$.

**Q6.** *Answers*: (a) divergent to $\infty$. (b) convergent to $\sqrt{2}$. Note: two singular points for the integrand: $x = 0$ and $x = 2$.

**Q7.** *Answers*: (a). divergent, it is oscillatory between -1 and 1. (b). convergent to $-\frac{2}{5}$. (c). divergent to $\infty$.

**Q8.** *Answers*: (a). it is convergent shown by Ratio Test. (b). it is convergent shown by the Comparison Test with a $p$-series for $p = 3$. (c). it is convergent shown by the Alternating Test.

**Q9.** *Answers*: (a). $\frac{1}{6}$. (b). [0,4]. (c). $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n+1)!}$.