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Structural stability of subsonic steady-states to the bipolar Euler-Poisson equations with degenerate boundary

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Abstract

This paper concerns the structural stability of subsonic steady-states to the bipolar Euler-Poisson equations under small perturbation of doping profiles. Here, the electron density is imposed with degenerate sonic boundary and considered in interiorly subsonic case, while the hole density is considered in fully subsonic case. Unlike the unipolar model, we show that the structural stability in bipolar model holds regardless of the type of doping profile and propose a new version of comparison principle, which captures the intrinsic negative correlation between electrons and holes. To overcome the difficulty caused by the degenerate effect at the sonic boundary, we introduce two different weight functions to handle the singularity near both sides of endpoints separately. The approaches adopted to prove the structural stability include the local singularity analysis, the monotonicity method, the continuation argument and the squeezing skill. Several numerical simulations are performed which support our theoretical results.

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Keywords: Bipolar Euler-Poisson equations; Subsonic steady-states; Structural stability; Degenerate boundary

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1. Introduction and main results

The bipolar Euler-Poisson equations arise from the modeling of semiconductor devices composed of electrons and holes (see [5,25]). The one-dimensional time-dependent bipolar Euler-Poisson equations are given by

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + \Big(\rho u^2 + p_1(\rho)\Big)_x = \rho E, \\ n_t + (nv)_x = 0, \\ (nv)_t + \Big(nv^2 + p_2(n)\Big)_x = -nE, \\ E_x = \rho - n - b(x). \end{cases} \tag{1.1}$$

Here ρ, n, u, v and E denote the electron density, the hole density, the electron velocity, the hole velocity and the electric field, respectively. The known functions $p_1(\rho)$ and $p_2(n)$ represent the pressure of electron and the pressure of hole, respectively. In this paper, for isothermal flow, we assume

$$p_1(\rho) = T\rho, \quad p_2(n) = Tn$$

with a constant temperature $T > 0$. The given function $b(x) > 0$ is the doping profile, which stands for the density of impurities in semiconductor devices.

The main purpose of this paper is to study the following steady-state equations of (1.1):

$$\begin{cases} J_1 = \text{constant}_1, \\ \left(\dfrac{J_1^2}{\rho} + T\rho\right)_x = \rho E, \\ J_2 = \text{constant}_2, \\ \left(\dfrac{J_2^2}{n} + Tn\right)_x = -nE, \\ E_x = \rho - n - b(x), \end{cases} \tag{1.2}$$

where $J_1 := \rho u$ is the current density of electron and $J_2 := nv$ is the current density of hole. Without loss of generality, we set $T = 1$, $J_1 = J$ and $J_2 = -J$, where $J > 0$ is a constant. Therefore, (1.2) can be reduced to

$$\begin{cases} \left(1 - \dfrac{J^2}{\rho^2}\right)\rho_x = \rho E, \\ \left(1 - \dfrac{J^2}{n^2}\right)n_x = -nE, \\ E_x = \rho - n - b(x). \end{cases} \tag{1.3}$$

By the terminology in gas dynamics, we call $c_e := \sqrt{p_1'(\rho)} = \sqrt{T} = 1$ the sound speed of electron and $c_h := \sqrt{p_2'(n)} = \sqrt{T} = 1$ the sound speed of hole. The electron flow is said to be subsonic/sonic/supersonic if

$$|u| = \frac{|J_1|}{\rho} = \frac{J}{\rho} \lesseqqgtr c_e = 1, \quad \text{or equivalently,} \quad \rho \gtreqqless J,$$

and the hole flow is said to be subsonic/sonic/supersonic if

$$|v| = \frac{|J_2|}{n} = \frac{J}{n} \lesseqqgtr c_h = 1, \quad \text{or equivalently,} \quad n \gtreqqless J.$$

Throughout this paper, we consider (1.3) on a bounded interval [0, 1]. The critical sonic boundary condition for the electron density and a given subsonic boundary condition for the hole density are proposed as follows:

$$\rho(0) = \rho(1) = J, \quad n(0) = \sigma_0 > J. \tag{1.4}$$

We also assume that the doping profile $b(x) \in C[0, 1]$. For simplicity of notation, we denote

$$\underline{b} := \inf_{x\in[0,1]} b(x) \quad \text{and} \quad \overline{b} := \sup_{x\in[0,1]} b(x).$$

Background of study. Over the past few decades, the study of the stationary Euler-Poisson equations has been a fascinating subject. The research of the unipolar stationary Euler-Poisson equations was initiated by Degond-Markowich [11]. For one-dimensional case, they proved the existence and uniqueness of purely subsonic solution under a smallness assumption on the current density. Since then, many facets of subsonic steady-states have been extensively investigated, see [3,4,12,27,29,30,38], for instance. For the supersonic steady flows, Peng-Violet [33] showed the existence and uniqueness of one-dimensional supersonic solution corresponding to large current density. Bae-Duan-Xiao-Xie [2] established the well-posedness of supersonic solution in a two-dimensional domain with a rectangular geometry. As regards the transonic case, Ascher-Markowich-Pietra-Schmeiser [1] first investigated the one-dimensional transonic steady-states with subsonic boundary conditions and a constant supersonic doping profile via phase-plane analysis. Gamba and Morawetz [16,17] constructed the one-dimensional and two-dimensional transonic solutions with shocks by artificial viscosity approximation, but the solutions, as the limits of vanishing viscosity, yield boundary layers. Under different boundary settings, Luo-Xin [24] made significant progress on the detailed structure to the one-dimensional model, establishing the existence/non-existence and the uniqueness/non-uniqueness of transonic solutions. Further, Luo-Rauch-Xie-Xin [23] proved the structural and dynamical stabilities of steady transonic shocks. Wei-Mei-Zhang-Zhang [37] investigated the smooth transonic solutions and observed that the crucial mechanism affecting the structure of the stationary Euler-Poisson equations is the doping profile.

The results mentioned above are related to non-degenerate states. When the boundary is subject to be sonic, the structure of physical solutions to the stationary Euler-Poisson equations becomes more complicated due to the strong singularity caused by the degenerate effect at the critical boundary. Li-Mei-Zhang-Zhang [21,22] first studied the one-dimensional unipolar Euler-Poisson equations with sonic boundary condition. They systematically classified the structure of subsonic/supersonic/transonic solutions when the doping profile is subsonic/supersonic. Subsequently, Chen-Mei-Zhang-Zhang extended the results of [21,22] to the radially symmetric spiral flows in [7], the transonic doping profile case in [8] and the multi-dimensional case in [9,10]. Very recently, Feng-Hu-Mei [14] and Feng-Mei-Zhang [15] established the structural stability of subsonic solution and the structural stability of smooth transonic solution, respectively. Xu-Mei-Nishibata [39] studied the structural stability of subsonic solution in multi-dimensional case.

Despite the great significance in physical practice, the mathematical results of bipolar stationary Euler-Poisson equations are few and limited due to the coupling of electrons and holes. Zhou-Li [41] proved the existence and uniqueness of stationary solution with Ohmic contact boundary conditions when the doping profile is zero. Tsuge [36] took account of the non-flat doping profile and obtained the existence and uniqueness of purely subsonic solution when the electrostatic potential is small enough. Yu [40] showed the existence and uniqueness of one-dimensional and two-dimensional subsonic solutions with insulating boundary conditions by the calculus of variations. Lately, based on the topological degree method, Mu-Mei-Zhang [28] investigated the existence and non-existence of several types of stationary solutions with respect to degenerate sonic boundary condition for the electron density and different non-degenerate boundary conditions for the hole density.

For more discussions on the stationary Euler-Poisson equations concerning time-asymptotic behavior of solutions and asymptotic limits of small parameters, we refer to [6,13,18–20,26,31,32,34] and references therein.

Feature and difficulty of our study. In practical applications, the conductivity of semiconductors can be improved by adding an appropriate amount of impurities (see [25]). The doping profile represents the density of impurities in semiconductor devices. As shown in [8,21,22,37], from the mathematical viewpoint, the doping profile plays a major role in the well-posedness/ill-posedness of physical solutions to the stationary Euler-Poisson equations. Therefore, it is of significance to investigate the structural stability of the steady-states, that is, under small perturbation of doping profiles, we expect the difference between the corresponding solutions to be small. In [23], Luo-Rauch-Xie-Xin first considered this topic regarding the unipolar stationary Euler-Poisson equations and proved the structural stability of transonic shocks with transonic boundary. In their settings, no singularity appears on the sonic line due to the fact that the transonic shocks jump the sonic line without interaction. For the structural stability of smooth transonic solution with transonic boundary, the singularity arises when the transonic solution cross the sonic line. Feng-Mei-Zhang [15] successfully addressed this situation by some technical analysis around the singular points. In the case of sonic boundary, the steady-states exhibit singularity at the degenerate boundary. Under the assumptions of subsonic doping profile and small momentum relaxation time, Feng-Hu-Mei [14] established the structural stability of subsonic steady-states to the one-dimensional unipolar Euler-Poisson equations with relaxation term by precise weighted multiplier method and monotonicity argument.

However, the structural stability of steady-states to the bipolar Euler-Poisson equations is still unknown. The aim of this paper is to establish the structural stability of subsonic steady-states to (1.3)-(1.4). There are some essential difficulties in this study. Owing to the strong singularity caused by the boundary degeneracy and the coupling of electrons and holes, it is full of challenges to derive the estimates near boundary points $x = 0$ and $x = 1$. Further, it is difficult to deal with the electric field in the stationary Euler-Poisson equations. For unipolar Euler-Poisson equations with subsonic doping profile, relaxation term and sonic boundary, according to the phase-plane analysis in [21], the value of the electric field at $x = 0$ is the reciprocal of the relaxation time. Unfortunately, the bipolar system (1.3)-(1.4) does not contain relaxation term and the value of the electric field at $x = 0$ is vague. Therefore, the existing approaches for unipolar model are ineffective in bipolar case.

In order to overcome the above difficulties, we propose some new ideas for the proof. We establish a new version of comparison principle, which captures the intrinsic negative correlation between electrons and holes. Moreover, we introduce the $x^{\frac{1}{2}}$-weight function to control the singularity at $x = 0$ and the $(1 - x)^{\frac{1}{2}}$-weight function to control the singularity at $x = 1$. The local weighted singularity analysis, the monotonicity method, the continuation argument and the squeezing skill are introduced to deal with the coupling of electrons and holes and the degeneracy of electrons at the critical sonic boundary. It is worth mentioning that the perturbation of electric field at each endpoint can be controlled by the perturbation of doping profiles, which is the core of establishing the structural stability.

Before proceeding, we first give the definition of the subsonic solution to (1.3)-(1.4). Due to the degeneracy of the system at the sonic boundary, the subsonic solution has to be defined in the weak sense, as in [21,28].

Definition 1.1. $(\rho, n)(x)$ is said to be a subsonic solution to (1.3)-(1.4) if

(i) $(\rho - J)^2 \in H_0^1(0, 1)$, $n \in W^{2,\infty}(0, 1)$;
(ii) $\rho(x) > J$ for $x \in (0, 1)$, $n(x) > J$ for $x \in [0, 1]$;
(iii) $\rho(0) = \rho(1) = J$, $n(0) = \sigma_0 > J$;
(iv) For any $\varphi \in H_0^1(0, 1)$, one has

$$\int_0^1 \left(\frac{1}{\rho} - \frac{J^2}{\rho^3}\right) \rho_x \varphi_x dx + \int_0^1 (\rho - n - b)\, \varphi dx = 0,$$

and

$$\int_0^1 \left(\frac{1}{n} - \frac{J^2}{n^3}\right) n_x \varphi_x dx + \int_0^1 (n + b - \rho)\, \varphi dx = 0.$$

Remark 1.1. Once $(\rho, n)(x)$ is determined, in view of $(1.3)_1$, $E(x)$ can be solved by

$$E(x) = \left(\frac{1}{\rho} - \frac{J^2}{\rho^3}\right) \rho_x = \frac{(\rho + J)[(\rho - J)^2]_x}{2\rho^3}.$$

In this sense, $(\rho, n, E)(x)$ is also called a subsonic solution to (1.3)-(1.4).

We recall the existence of subsonic solution to (1.3)-(1.4) as follows, excerpted from [28].

Proposition 1.1 *([28]). Suppose that the doping profile $b \in L^\infty(0, 1)$. Then for any $\underline{n} > J$, there exists a constant $\sigma^* = \sigma^*(\bar{b}, \underline{n}) > J$ which only depends on $\bar{b}$ and $\underline{n}$, such that for any $\sigma_0 \geq \sigma^*$, the boundary value problem (1.3)-(1.4) admits a subsonic solution $(\rho, n, E) \in C^{\frac{1}{2}}[0, 1] \times W^{2,\infty}(0, 1) \times H^1(0, 1)$.*

Remark 1.2. In Proposition 1.1, for a doping profile $b \in C[0,1]$, by the interior regularity theory of elliptic equations and Sobolev's embedding theorem, we obtain a subsonic solution $(\rho, n, E) \in \left(C^1(0,1) \cap C^{\frac{1}{2}}[0,1]\right) \times C^{1,1}[0,1] \times C^1[0,1]$ with relatively higher-order regularity.

Our main results are as follows.

Theorem 1.1 *(The uniqueness of subsonic solution). Suppose that the doping profile $b \in C[0,1]$. Then the system (1.3)-(1.4) admits a unique subsonic solution $(\rho, n, E)(x)$.*

Theorem 1.2 *(The structural stability of subsonic solution). Suppose that the doping profiles $b_1, b_2 \in C[0,1]$. For $i = 1, 2$, let $(\rho_i, n_i, E_i)(x)$ denote the subsonic solution to (1.3)-(1.4) corresponding to $b_i(x)$. Then $(\rho_1, n_1, E_1)(x)$ and $(\rho_2, n_2, E_2)(x)$ are structurally stable, namely, there exist two constants $\delta_0 \in (0, \frac{1}{2})$ and $C > 0$ independent of $\|b_1 - b_2\|_{C[0,1]}$ such that*

$$\begin{aligned} &\|\rho_1 - \rho_2\|_{C[0,1]} + \|x^{\frac{1}{2}}(\rho_1 - \rho_2)_x\|_{C[0,\delta_0)} + \|(\rho_1 - \rho_2)_x\|_{C[\delta_0, 1-\delta_0]} \\ &\quad + \|(1-x)^{\frac{1}{2}}(\rho_1 - \rho_2)_x\|_{C(1-\delta_0, 1]} \\ &\quad + \|n_1 - n_2\|_{C^1[0,1]} + \|E_1 - E_2\|_{C^1[0,1]} \leq C\|b_1 - b_2\|_{C[0,1]}. \end{aligned} \tag{1.5}$$

Remark 1.3. Unlike the unipolar case [14], in which an extra subsonic restriction on doping profiles is proposed, we prove the structural stability in bipolar case regardless of the type of doping profile. The negative correlation between electrons and holes leads to this essential difference.

Remark 1.4. In a similar way, the corresponding results in this paper can be extended to the isentropic case.

This paper is organized as follows. In Section 2, we show the uniqueness of subsonic solution to (1.3)-(1.4). Section 3 is devoted to establishing the structural stability of subsonic solution to (1.3)-(1.4). Finally, Section 4 presents some numerical simulations which support our theoretical results.

2. The uniqueness of subsonic solution

In this section, as a prerequisite for the structural stability, we prove the uniqueness of subsonic solution to (1.3)-(1.4).

Proof of Theorem 1.1. Assume that $(\rho^{(1)}, n^{(1)}, E^{(1)})$ and $(\rho^{(2)}, n^{(2)}, E^{(2)})$ are two subsonic solutions to (1.3)-(1.4). Therefore, for $i = 1, 2$, $(\rho^{(i)}, n^{(i)}, E^{(i)})$ satisfy

$$\begin{cases} \left(\dfrac{1}{\rho^{(i)}} - \dfrac{J^2}{(\rho^{(i)})^3}\right)(\rho^{(i)})_x = E^{(i)}, \\ \left(\dfrac{1}{n^{(i)}} - \dfrac{J^2}{(n^{(i)})^3}\right)(n^{(i)})_x = -E^{(i)}, & x \in (0,1), \\ (E^{(i)})_x = \rho^{(i)} - n^{(i)} - b(x), \\ \rho^{(i)}(0) = \rho^{(i)}(1) = J, \ n^{(i)}(0) = \sigma_0 > J, \end{cases} \tag{2.1}$$

and the equivalent form

$$\begin{cases} \left(w(\rho^{(i)})\right)_{xx} = \rho^{(i)} - n^{(i)} - b(x), \\ \left(\dfrac{1}{n^{(i)}} - \dfrac{J^2}{(n^{(i)})^3}\right)(n^{(i)})_x = -E^{(i)}, & x \in (0,1), \\ \rho^{(i)}(0) = \rho^{(i)}(1) = J,\ n^{(i)}(0) = \sigma_0 > J, \end{cases} \tag{2.2}$$

where

$$w(s) := \log s + \frac{J^2}{2s^2} \tag{2.3}$$

is increasing on $s \in [J, \infty)$. Taking the difference of $(2.2)_1|_{i=1}$ and $(2.2)_1|_{i=2}$ implies that

$$\left(w(\rho^{(1)}) - w(\rho^{(2)})\right)_{xx} = (\rho^{(1)} - \rho^{(2)}) - (n^{(1)} - n^{(2)}). \tag{2.4}$$

Then, for any $\varphi \in H_0^1(0,1)$, $\varphi \geq 0$, we get

$$-\int_0^1 \left(w(\rho^{(1)}) - w(\rho^{(2)})\right)_x \varphi_x dx = \int_0^1 (\rho^{(1)} - \rho^{(2)})\varphi dx - \int_0^1 (n^{(1)} - n^{(2)})\varphi dx. \tag{2.5}$$

Taking $\varphi = \left(w(\rho^{(1)}) - w(\rho^{(2)})\right)^+ = \max\left\{0, w(\rho^{(1)}) - w(\rho^{(2)})\right\}$ in (2.5) gives

$$\begin{aligned} &-\int_0^1 \left|\left(w(\rho^{(1)}) - w(\rho^{(2)})\right)_x^+\right|^2 dx \\ &= \int_0^1 (\rho^{(1)} - \rho^{(2)}) \left(w(\rho^{(1)}) - w(\rho^{(2)})\right)^+ dx - \int_0^1 (n^{(1)} - n^{(2)}) \left(w(\rho^{(1)}) - w(\rho^{(2)})\right)^+ dx. \end{aligned} \tag{2.6}$$

According to the monotonicity of w, we obtain

$$\int_0^1 (\rho^{(1)} - \rho^{(2)}) \left(w(\rho^{(1)}) - w(\rho^{(2)})\right)^+ dx \geq 0. \tag{2.7}$$

Summing up $(2.1)_1$ and $(2.1)_2$, we have

$$\left(w(\rho^{(i)}) + w(n^{(i)})\right)_x = 0, \quad i = 1, 2. \tag{2.8}$$

For any $x \in (0, 1]$, integrating (2.8) over $(0, x)$, we deduce

$$w(\rho^{(i)}) - \left(\log J + \frac{1}{2}\right) + w(n^{(i)}) - \left(\log \sigma_0 + \frac{J^2}{2\sigma_0^2}\right) = 0, \quad i = 1, 2. \tag{2.9}$$

Taking the difference of (2.9)$|_{i=1}$ and (2.9)$|_{i=2}$ implies that

$$w(\rho^{(1)}) - w(\rho^{(2)}) = w(n^{(2)}) - w(n^{(1)}). \tag{2.10}$$

It is worth noticing that (2.10) shows the negative correlation between electrons and holes, which plays a crucial role in the following proof. Hence, the monotonicity of w yields

$$-\int_0^1 (n^{(1)} - n^{(2)}) \left(w(\rho^{(1)}) - w(\rho^{(2)})\right)^+ dx = \int_0^1 (n^{(2)} - n^{(1)}) \left(w(n^{(2)}) - w(n^{(1)})\right)^+ dx \geq 0. \tag{2.11}$$

Substituting (2.7) and (2.11) into (2.6), we have

$$\int_0^1 \left| \left(w(\rho^{(1)}) - w(\rho^{(2)})\right)_x^+ \right|^2 dx = 0. \tag{2.12}$$

(2.12) together with Poincaré's inequality leads to

$$\int_0^1 \left| \left(w(\rho^{(1)}) - w(\rho^{(2)})\right)^+ \right|^2 dx = 0. \tag{2.13}$$

Thus, we have $w(\rho^{(1)}) \leq w(\rho^{(2)})$. Using the monotonicity of w again, we get $\rho^{(1)} \leq \rho^{(2)}$.

By a similar argument, taking $\varphi = \left(w(\rho^{(1)}) - w(\rho^{(2)})\right)^- = -\min\left\{0, w(\rho^{(1)}) - w(\rho^{(2)})\right\}$ in (2.5) yields $\rho^{(1)} \geq \rho^{(2)}$ by repeating the above process. Consequently, $\rho^{(1)} = \rho^{(2)}$.

Moreover, (2.10) and the monotonicity of w yield $n^{(1)} = n^{(2)}$. Additionally, by $(2.1)_2$, we obtain $E^{(1)} = E^{(2)}$. The proof of uniqueness is complete.

3. The structural stability of subsonic solution

This section aims to prove the structural stability of subsonic solution to (1.3)-(1.4). The analysis carried out is long and technical. We divide the proof into several lemmas.

For $i = 1, 2$, let $(\rho_i, n_i, E_i)(x)$ denote the subsonic solution to (1.3)-(1.4) corresponding to $b_i(x)$, then $(\rho_i, n_i, E_i)(x)$ satisfies

$$\begin{cases} \left(1 - \dfrac{J^2}{(\rho_i)^2}\right)(\rho_i)_x = \rho_i E_i, \\ \left(1 - \dfrac{J^2}{(n_i)^2}\right)(n_i)_x = -n_i E_i, \\ (E_i)_x = \rho_i - n_i - b_i(x), \\ \rho_i(0) = \rho_i(1) = J, \; n_i(0) = \sigma_0 > J. \end{cases} \quad x \in (0, 1). \tag{3.1}$$

The degeneracy of (3.1) at the sonic boundary $\rho_i(0) = \rho_i(1) = J$ will bring multiple difficulties to the proof of the structural stability. We begin with analyzing the sign of $E_i(0)$ and the boundary behaviors of $\rho_i(x)$ and $(\rho_i)_x(x)$ near the left endpoint $x = 0$.

Lemma 3.1. *For $i = 1, 2$, let $b_i \in C[0, 1]$. Then,*

$$E_i(0) > 0, \tag{3.2}$$

and there exist positive constants C_j $(j = 1, 2, 3, 4)$ such that, for x near the left endpoint 0,

$$C_1 x^{\frac{1}{2}} \le \rho_i(x) - J \le C_2 x^{\frac{1}{2}}, \tag{3.3}$$

and

$$C_3 x^{-\frac{1}{2}} \le (\rho_i)_x(x) \le C_4 x^{-\frac{1}{2}}, \tag{3.4}$$

where $C_2 > C_1 > 0$ and $C_4 > C_3 > 0$.

Proof. For $i = 1, 2$, let us first prove $E_i(0) > 0$. Otherwise, we assume that $E_i(0) \le 0$ by contradiction. According to Remark 1.2, we have $(\rho_i, n_i) \in C[0, 1] \times C^{1,1}[0, 1]$. Therefore, there exist constants $\bar{\rho} > J$ and $\bar{n} > \underline{n} > J$ such that

$$J \le \rho_i(x) \le \bar{\rho}, \quad J < \underline{n} \le n_i(x) \le \bar{n}, \quad x \in [0, 1]. \tag{3.5}$$

Since $n_i(0) = \sigma_0 > \rho_i(0) = J$ and $b_i(x) > 0$ for $x \in [0, 1]$, there exists a constant $\varepsilon \in (0, \frac{1}{2})$ such that $\rho_i(x) - n_i(x) - b_i(x) < 0$ for $x \in [0, \varepsilon]$. Integrating $(3.1)_3$ over $(0, x)$ for $x \in (0, \varepsilon]$ yields

$$E_i(x) = E_i(0) + \int_0^x [\rho_i(s) - n_i(s) - b_i(s)]\, ds < E_i(0) \le 0.$$

This together with $(3.1)_1$ implies $(\rho_i)_x(x) < 0$ on $(0, \varepsilon)$, which is in contradiction with $\rho_i(0) = J$ and $\rho_i(x) > J$ over $(0, 1)$.

By the continuity of $E_i(x)$, there exists a constant $\kappa \in (0, \varepsilon)$ such that

$$0 < \frac{E_i(0)}{2} \le E_i(x) \le \frac{3E_i(0)}{2}, \quad x \in [0, \kappa]. \tag{3.6}$$

By $(3.1)_1$, we have

$$E_i(x) = \frac{(\rho_i)^2 - J^2}{(\rho_i)^3} (\rho_i)_x = \frac{(\rho_i + J)\left[(\rho_i - J)^2\right]_x}{2(\rho_i)^3}. \tag{3.7}$$

Inserting (3.5) and (3.6) into (3.7), for $x \in [0, \kappa]$, we get

$$0 < \frac{J^3 E_i(0)}{\bar{\rho} + J} \le \frac{(\rho_i)^3 E_i(0)}{\rho_i + J} \le \left[(\rho_i - J)^2\right]_x = \frac{2(\rho_i)^3 E_i(x)}{\rho_i + J} \le \frac{3(\rho_i)^3 E_i(0)}{\rho_i + J} \le \frac{3\bar{\rho}^3 E_i(0)}{2J}. \tag{3.8}$$

Integrating (3.8) over $(0, x)$ for $x \in (0, \kappa]$, we deduce

$$C_1 x^{\frac{1}{2}} \le \rho_i(x) - J \le C_2 x^{\frac{1}{2}}, \quad x \in [0, \kappa], \tag{3.9}$$

with

$$C_1 := \min\left\{\sqrt{\frac{J^3 E_1(0)}{\bar{\rho} + J}}, \sqrt{\frac{J^3 E_2(0)}{\bar{\rho} + J}}\right\}, \quad C_2 := \max\left\{\sqrt{\frac{3\bar{\rho}^3 E_1(0)}{2J}}, \sqrt{\frac{3\bar{\rho}^3 E_2(0)}{2J}}\right\}.$$

Furthermore, it follows from (3.8) that

$$\frac{J^3 E_i(0)}{2(\bar{\rho} + J)(\rho_i - J)} \le (\rho_i)_x \le \frac{3\bar{\rho}^3 E_i(0)}{4J(\rho_i - J)}, \quad x \in [0, \kappa]. \tag{3.10}$$

Substituting (3.9) into (3.10), we obtain

$$C_3 x^{-\frac{1}{2}} \le (\rho_i)_x(x) \le C_4 x^{-\frac{1}{2}}, \quad x \in [0, \kappa],$$

with

$$C_3 := \min\left\{\frac{J^3 E_1(0)}{2(\bar{\rho} + J)C_2}, \frac{J^3 E_2(0)}{2(\bar{\rho} + J)C_2}\right\}, \quad C_4 := \max\left\{\frac{3\bar{\rho}^3 E_1(0)}{4JC_1}, \frac{3\bar{\rho}^3 E_2(0)}{4JC_1}\right\}.$$

This completes the proof of Lemma 3.1. □

Following a similar reasoning in the proof of Lemma 3.1, for $i = 1, 2$, we can determine the sign of $E_i(1)$ and the boundary behaviors of $\rho_i(x)$ and $(\rho_i)_x(x)$ near the right endpoint $x = 1$. Details are skipped.

Lemma 3.2. *For $i = 1, 2$, let $b_i \in C[0, 1]$. Then,*

$$E_i(1) < 0, \tag{3.11}$$

and there exist positive constants C_j $(j = 5, 6, 7, 8)$ such that, for x near the right endpoint 1,

$$C_5(1 - x)^{\frac{1}{2}} \le \rho_i(x) - J \le C_6(1 - x)^{\frac{1}{2}}, \tag{3.12}$$

and

$$-C_7(1 - x)^{-\frac{1}{2}} \le (\rho_i)_x(x) \le -C_8(1 - x)^{-\frac{1}{2}}, \tag{3.13}$$

where $C_6 > C_5 > 0$ and $C_7 > C_8 > 0$.

We observe from (3.4) and (3.13) that the first order derivative of $\rho_i(x)$ blows up near $x = 0$ and $x = 1$, namely,

$$\lim_{x\to 0^+} (\rho_i)_x(x) = +\infty, \quad \lim_{x\to 1^-} (\rho_i)_x(x) = -\infty. \tag{3.14}$$

Therefore, endpoints $x = 0$ and $x = 1$ are both singular points. In order to deal with the singularity of the solution at the sonic boundary, we introduce the $x^{\frac{1}{2}}$-weight function to control the singularity at $x = 0$ and the $(1-x)^{\frac{1}{2}}$-weight function to control the singularity at $x = 1$ in order to perform the following local weighted singularity analysis.

Lemma 3.3. *For $i = 1, 2$, one has*

$$\lim_{x\to 0^+} x^{\frac{1}{2}} (\rho_i)_x(x) = \frac{J}{2}\sqrt{E_i(0)} > 0, \tag{3.15}$$

and

$$\lim_{x\to 1^-} (1-x)^{\frac{1}{2}} (\rho_i)_x(x) = -\frac{J}{2}\sqrt{-E_i(1)} < 0. \tag{3.16}$$

Proof. It follows from (3.3) and (3.12) that the coefficient $1 - \frac{J^2}{(\rho_i)^2}$ in the degenerate principal part of $(3.1)_1$ is comparable to $x^{\frac{1}{2}}$ near the left endpoint $x = 0$ and $(1-x)^{\frac{1}{2}}$ near the right endpoint $x = 1$. By the regularity theory of boundary degenerate elliptic equations [35], we know that $x^{\frac{1}{2}}(\rho_i)_x(x)$ can be continuous up to $x = 0$ and $(1-x)^{\frac{1}{2}}(\rho_i)_x(x)$ can be continuous up to $x = 1$.

Let us first calculate $\lim_{x\to 0^+} x^{\frac{1}{2}}(\rho_i)_x(x)$. For convenience, we set

$$A_i := \lim_{x\to 0^+} x^{\frac{1}{2}} (\rho_i)_x(x).$$

Multiplying $(3.1)_1$ by $x^{\frac{1}{2}} \frac{(\rho_i)^2}{(\rho_i)^2 - J^2}$ yields

$$x^{\frac{1}{2}} (\rho_i)_x = \frac{(\rho_i)^3 E_i}{\rho_i + J} \frac{x^{\frac{1}{2}}}{\rho_i - J}.$$

Bearing in mind $\rho_i(0) = J$ and using L'Hospital's rule, we have

$$\begin{aligned} A_i &= \lim_{x\to 0^+} x^{\frac{1}{2}} (\rho_i)_x \\ &= \lim_{x\to 0^+} \frac{(\rho_i)^3}{\rho_i + J} \lim_{x\to 0^+} E_i \lim_{x\to 0^+} \frac{x^{\frac{1}{2}}}{\rho_i - J} \\ &= \frac{J^2}{2} E_i(0) \lim_{x\to 0^+} \frac{\frac{1}{2}x^{-\frac{1}{2}}}{(\rho_i)_x} \\ &= \frac{J^2}{4} E_i(0) \frac{1}{A_i}. \end{aligned}$$

The boundary behavior (3.4) prompts us to choose the positive root

$$A_i = \frac{J}{2}\sqrt{E_i(0)} > 0.$$

Next, we compute $\lim_{x\to 1^-}(1-x)^{\frac{1}{2}}(\rho_i)_x(x)$. Set

$$B_i := \lim_{x\to 1^-}(1-x)^{\frac{1}{2}}(\rho_i)_x(x).$$

Multiplying $(3.1)_1$ by $(1-x)^{\frac{1}{2}}\frac{(\rho_i)^2}{(\rho_i)^2-J^2}$ gives

$$(1-x)^{\frac{1}{2}}(\rho_i)_x = \frac{(\rho_i)^3 E_i}{\rho_i+J}\frac{(1-x)^{\frac{1}{2}}}{\rho_i-J}.$$

Bearing in mind $\rho_i(1)=J$ and using L'Hospital's rule, we obtain

$$\begin{aligned} B_i &= \lim_{x\to 1^-}(1-x)^{\frac{1}{2}}(\rho_i)_x \\ &= \lim_{x\to 1^-}\frac{(\rho_i)^3}{\rho_i+J}\lim_{x\to 1^-}E_i\lim_{x\to 1^-}\frac{(1-x)^{\frac{1}{2}}}{\rho_i-J} \\ &= \frac{J^2}{2}E_i(1)\lim_{x\to 1^-}\frac{-\frac{1}{2}(1-x)^{-\frac{1}{2}}}{(\rho_i)_x} \\ &= -\frac{J^2}{4}E_i(1)\frac{1}{B_i}. \end{aligned}$$

This together with the boundary behavior (3.13) implies that

$$B_i = -\frac{J}{2}\sqrt{-E_i(1)} < 0,$$

which is negative. The proof of Lemma 3.3 is complete. □

Next, we establish the key comparison principle for the subsonic solution, which is the cornerstone of monotonicity argument in proving structural stability.

Lemma 3.4 *(Comparison principle). Let $b_1, b_2 \in C[0,1]$. If $b_1(x) \geq b_2(x)$ on $[0,1]$, then*

$$\rho_1(x) \geq \rho_2(x), \quad n_1(x) \leq n_2(x), \qquad \textit{for } x \in [0,1]. \tag{3.17}$$

Proof. For $i = 1, 2$, let $(\rho_i, n_i, E_i)(x)$ denote the subsonic solution to (3.1) corresponding to $b_i(x)$. By arguing similarly as in the proof of Theorem 1.1, we obtain

$$(w(\rho_1)-w(\rho_2))_{xx} = (\rho_1-\rho_2)-(n_1-n_2)-(b_1-b_2), \tag{3.18}$$

and

$$w(\rho_1) - w(\rho_2) = w(n_2) - w(n_1), \tag{3.19}$$

where w is defined in (2.3). Multiplying (3.18) by $(w(\rho_2) - w(\rho_1))^+$ and integrating the resulting equation over $(0, 1)$ yield

$$\begin{aligned}
&\int_0^1 \left|(w(\rho_2) - w(\rho_1))_x^+\right|^2 dx + \int_0^1 (\rho_2 - \rho_1)(w(\rho_2) - w(\rho_1))^+ dx \\
&\quad - \int_0^1 (n_2 - n_1)(w(\rho_2) - w(\rho_1))^+ dx \\
&\quad = \int_0^1 (b_2 - b_1)(w(\rho_2) - w(\rho_1))^+ dx \le 0.
\end{aligned} \tag{3.20}$$

Bearing in mind (3.19) and the monotonicity of w, we get

$$\begin{aligned}
&\int_0^1 (\rho_2 - \rho_1)(w(\rho_2) - w(\rho_1))^+ dx - \int_0^1 (n_2 - n_1)(w(\rho_2) - w(\rho_1))^+ dx \\
&= \int_0^1 (\rho_2 - \rho_1)(w(\rho_2) - w(\rho_1))^+ dx + \int_0^1 (n_1 - n_2)(w(n_1) - w(n_2))^+ dx \ge 0.
\end{aligned} \tag{3.21}$$

Inserting (3.21) into (3.20) and using Poincaré's inequality imply

$$0 \le \int_0^1 \left|(w(\rho_2) - w(\rho_1))^+\right|^2 dx \le \int_0^1 \left|(w(\rho_2) - w(\rho_1))_x^+\right|^2 dx \le 0. \tag{3.22}$$

Therefore, we have $w(\rho_1) \ge w(\rho_2)$. This together with the monotonicity of w and (3.19) gives $\rho_1(x) \ge \rho_2(x)$ and $n_1(x) \le n_2(x)$ for $x \in [0, 1]$. □

Lemmas 3.1-3.4 enable us to establish the structural stability of subsonic solution to (3.1) at the cost of adding an extra restriction $b_1(x) \ge b_2(x)$. Since it is difficult to study the structural stability of the solution directly on the entire interval $[0, 1]$, we divide the interval $[0, 1]$ into three parts: (i) near the left endpoint $x = 0$; (ii) on the middle domain; (iii) near the right endpoint $x = 1$. We begin with the following lemma which establishes the local weighted structural stability estimate on a neighborhood of the left endpoint $x = 0$.

Lemma 3.5 *(Local weighted structural stability estimate near $x = 0$). Let $b_1, b_2 \in C[0, 1]$ and $b_1(x) \ge b_2(x)$ on $[0, 1]$. Then there exist two constants $\delta_0 \in (0, \frac{1}{2})$ and $C > 0$ independent of $\|b_1 - b_2\|_{C[0,1]}$ such that*

$$\begin{aligned}&\|x^{-\frac{1}{2}}(\rho_1-\rho_2)\|_{C[0,\delta_0)}+\|x^{\frac{1}{2}}(\rho_1-\rho_2)_x\|_{C[0,\delta_0)}\\&\quad+\|n_1-n_2\|_{C^1[0,\delta_0)}+\|E_1-E_2\|_{C^1[0,\delta_0)}\le C\|b_1-b_2\|_{C[0,1]}.\end{aligned}\tag{3.23}$$

Proof. We first claim that

$$E_1(1)\le E_2(1)<0<E_2(0)\le E_1(0).\tag{3.24}$$

In fact, since $b_1(x)\ge b_2(x)$ on $[0,1]$, Lemma 3.4 gives $\rho_1(x)\ge\rho_2(x)$ on $[0,1]$. Bearing in mind $\rho_1(0)=\rho_2(0)=J$, it follows from L'Hospital's rule and (3.15) that

$$\begin{aligned}0\le&\lim_{x\to0^+}\frac{(\rho_1-\rho_2)(x)}{x^{\frac{1}{2}}}\\=&\lim_{x\to0^+}\frac{(\rho_1-J)(x)}{x^{\frac{1}{2}}}-\lim_{x\to0^+}\frac{(\rho_2-J)(x)}{x^{\frac{1}{2}}}\\=&2\lim_{x\to0^+}x^{\frac{1}{2}}(\rho_1)_x-2\lim_{x\to0^+}x^{\frac{1}{2}}(\rho_2)_x\\=&J\left(\sqrt{E_1(0)}-\sqrt{E_2(0)}\right).\end{aligned}\tag{3.25}$$

Thus, $E_1(0)\ge E_2(0)>0$. Similarly as above, combining $\rho_1(1)=\rho_2(1)=J$, L'Hospital's rule and (3.16) yields

$$\begin{aligned}0\le&\lim_{x\to1^-}\frac{(\rho_1-\rho_2)(x)}{(1-x)^{\frac{1}{2}}}\\=&\lim_{x\to1^-}\frac{(\rho_1-J)(x)}{(1-x)^{\frac{1}{2}}}-\lim_{x\to1^-}\frac{(\rho_2-J)(x)}{(1-x)^{\frac{1}{2}}}\\=&-2\lim_{x\to1^-}(1-x)^{\frac{1}{2}}(\rho_1)_x+2\lim_{x\to1^-}(1-x)^{\frac{1}{2}}(\rho_2)_x\\=&J\left(\sqrt{-E_1(1)}-\sqrt{-E_2(1)}\right),\end{aligned}\tag{3.26}$$

which implies $E_1(1)\le E_2(1)<0$. Therefore, we obtain (3.24).

Next, we show that there exist two constants $\delta_0\in(0,\frac{1}{2})$ and $M_0>0$ independent of $\|b_1-b_2\|_{C[0,1]}$ such that

$$\frac{(\rho_1-\rho_2)(x)}{x^{\frac{1}{2}}}\le M_0\|b_1-b_2\|_{C[0,1]},\quad x\in[0,\delta_0).\tag{3.27}$$

Otherwise, for any $\delta\in(0,\frac{1}{2})$ and $M>0$, there exists $x_\delta\in[0,\delta_0)$ such that

$$\frac{(\rho_1-\rho_2)(x_\delta)}{(x_\delta)^{\frac{1}{2}}}>M\|b_1-b_2\|_{C[0,1]}.\tag{3.28}$$

By the arbitrariness of δ, we could take $\delta=\frac{1}{h}$ with $h=3,4,5,\cdots$. Hence, for any $M>0$, there exists $x_h\in[0,\frac{1}{h})$ such that

$$\frac{(\rho_1-\rho_2)(x_h)}{(x_h)^{\frac{1}{2}}} > M\|b_1-b_2\|_{C[0,1]}, \tag{3.29}$$

which yields

$$\underline{\lim}_{x_h\to 0^+} \frac{(\rho_1-\rho_2)(x_h)}{(x_h)^{\frac{1}{2}}} \geq M\|b_1-b_2\|_{C[0,1]}. \tag{3.30}$$

Indeed, combining $(3.1)_3$, (3.24), (3.25) and Lemma 3.4, we deduce

$$\begin{aligned}
\lim_{x\to 0^+} \frac{(\rho_1-\rho_2)(x)}{x^{\frac{1}{2}}} &= J\left(\sqrt{E_1(0)}-\sqrt{E_2(0)}\right)\\
&= C_0\left(E_1(0)-E_2(0)\right)\\
&\leq C_0\Big((E_1(0)-E_1(1))-(E_2(0)-E_2(1))\Big)\\
&= C_0\left(\int_0^1 (b_1+n_1-\rho_1)dx - \int_0^1 (b_2+n_2-\rho_2)dx\right)\\
&= C_0\left(\int_0^1 (b_1-b_2)dx + \int_0^1 (n_1-n_2)dx - \int_0^1 (\rho_1-\rho_2)dx\right)\\
&\leq C_0\int_0^1 (b_1-b_2)dx\\
&\leq \widetilde{C}_0\|b_1-b_2\|_{C[0,1]},
\end{aligned} \tag{3.31}$$

where $C_0=\frac{J}{\sqrt{E_1(0)}+\sqrt{E_2(0)}}>0$ and $\widetilde{C}_0>0$ is a constant independent of $\|b_1-b_2\|_{C[0,1]}$. Thus, we get a contradiction to (3.30) by taking $M=2\widetilde{C}_0$, which implies (3.27). It is worth noticing that (3.31) gives

$$E_1(0)-E_2(0)\leq C\|b_1-b_2\|_{C[0,1]}, \tag{3.32}$$

which indicates that the difference between the values of electric field at $x=0$ can be controlled by the perturbation of doping profiles.

Bearing in mind $n_i\in C^{1,1}[0,1]$, the non-degeneracy of n_i and the smoothness of w defined in (2.3), by (3.19) and (3.27), we obtain

$$(n_2-n_1)(x)\leq C(w(n_2)-w(n_1)) = C(w(\rho_1)-w(\rho_2)) \leq \frac{C(\rho_1-\rho_2)}{x^{\frac{1}{2}}} \leq C\|b_1-b_2\|_{C[0,1]} \tag{3.33}$$

for $x\in[0,\delta_0)$.

With the preparations above, we are now in position to prove (3.23). By $(3.1)_1$, for $i=1,2$, we have

$$(\rho_i)_x = \frac{(\rho_i)^3 E_i}{(\rho_i)^2 - J^2}. \tag{3.34}$$

Multiplying (3.34) by $x^{\frac{1}{2}}$ and taking the difference of the resulting equations for $i = 1, 2$, we deduce

$$\begin{aligned} x^{\frac{1}{2}}(\rho_1 - \rho_2)_x &= \frac{(\rho_1)^3 E_1}{\rho_1 + J} \frac{x^{\frac{1}{2}}}{\rho_1 - J} - \frac{(\rho_2)^3 E_2}{\rho_2 + J} \frac{x^{\frac{1}{2}}}{\rho_2 - J} \\ &= \frac{(\rho_1)^3 E_1}{\rho_1 + J} \left(\frac{x^{\frac{1}{2}}}{\rho_1 - J} - \frac{x^{\frac{1}{2}}}{\rho_2 - J} \right) + \left(\frac{(\rho_1)^3 E_1}{\rho_1 + J} - \frac{(\rho_2)^3 E_2}{\rho_2 + J} \right) \frac{x^{\frac{1}{2}}}{\rho_2 - J} \\ &=: I_1 + I_2. \end{aligned} \tag{3.35}$$

Now, we estimate I_1 and I_2 near $x = 0$, respectively. It follows from (3.3) and (3.27) that

$$|I_1| = \left| \frac{(\rho_1)^3 E_1}{\rho_1 + J} \frac{x^{\frac{1}{2}}}{\rho_1 - J} \frac{x^{\frac{1}{2}}}{\rho_2 - J} \frac{\rho_2 - \rho_1}{x^{\frac{1}{2}}} \right| \le C \left(\frac{\rho_1 - \rho_2}{x^{\frac{1}{2}}} \right) \le C \|b_1 - b_2\|_{C[0,1]}. \tag{3.36}$$

Combining (3.3), (3.17), (3.24), (3.27), (3.32), (3.33) and the mean-value theorem implies that

$$\begin{aligned} |I_2| &= \left| \left(\frac{(\rho_1)^3 E_1}{\rho_1 + J} - \frac{(\rho_2)^3 E_2}{\rho_2 + J} \right) \frac{x^{\frac{1}{2}}}{\rho_2 - J} \right| \\ &\le C \left| \frac{(\rho_1)^3 E_1}{\rho_1 + J} - \frac{(\rho_2)^3 E_2}{\rho_2 + J} \right| \\ &= C \left| E_1 \left(\frac{(\rho_1)^3}{\rho_1 + J} - \frac{(\rho_2)^3}{\rho_2 + J} \right) + \frac{(\rho_2)^3}{\rho_2 + J} (E_1 - E_2) \right| \\ &\le C|\rho_1 - \rho_2|(x) + C \left| (E_1(0) - E_2(0)) + \int_0^x [(\rho_1 - \rho_2) - (n_1 - n_2) - (b_1 - b_2)]\, dy \right| \\ &\le C(\rho_1 - \rho_2)(x) + C(E_1(0) - E_2(0)) + C \int_0^x (\rho_1 - \rho_2) dy \\ &\quad + C \int_0^x (n_2 - n_1) dy + C\|b_1 - b_2\|_{C[0,1]} \\ &\le C \left(\frac{(\rho_1 - \rho_2)(x)}{x^{\frac{1}{2}}} + \frac{(\rho_1 - \rho_2)(\xi)}{\xi^{\frac{1}{2}}} \right) + C(E_1(0) - E_2(0)) \\ &\quad + C(n_2 - n_1)(\bar{\xi}) + C\|b_1 - b_2\|_{C[0,1]}, \quad \exists\, \xi, \bar{\xi} \in [0, x] \\ &\le C\|b_1 - b_2\|_{C[0,1]}. \end{aligned} \tag{3.37}$$

Inserting (3.36) and (3.37) into (3.35) yields

$$x^{\frac{1}{2}}|(\rho_1-\rho_2)_x|(x)\leq C\|b_1-b_2\|_{C[0,1]},\quad x\in[0,\delta_0). \tag{3.38}$$

By $(3.1)_2$, for $i=1,2$, we have

$$(n_i)_x=-\frac{(n_i)^3E_i}{(n_i)^2-J^2}. \tag{3.39}$$

Taking the difference of $(3.39)|_{i=1}$ and $(3.39)|_{i=2}$ and using the mean-value theorem, we get

$$\begin{aligned}(n_1-n_2)_x=&\frac{(n_2)^3E_2}{(n_2)^2-J^2}-\frac{(n_1)^3E_1}{(n_1)^2-J^2}\\=&-E_1\left(\frac{(n_1)^3}{(n_1)^2-J^2}-\frac{(n_2)^3}{(n_2)^2-J^2}\right)-\frac{(n_2)^3}{(n_2)^2-J^2}(E_1-E_2)\\=&-E_1f'(\zeta)(n_1-n_2)-f(n_2)(E_1-E_2),\quad\exists\,\zeta\in(n_1,n_2),\end{aligned} \tag{3.40}$$

where

$$f(s):=\frac{s^3}{s^2-J^2}. \tag{3.41}$$

In addition, taking the difference of $(3.1)_3|_{i=1}$ and $(3.1)_3|_{i=2}$ gives

$$(E_1-E_2)_x=(\rho_1-\rho_2)-(n_1-n_2)-(b_1-b_2). \tag{3.42}$$

Multiplying (3.40) by (n_1-n_2) and (3.42) by (E_1-E_2), using Cauchy's inequality and (3.27), we deduce

$$\begin{aligned}&\left((n_1-n_2)^2+(E_1-E_2)^2\right)_x(x)\\&\leq C\left((\rho_1-\rho_2)^2+(n_1-n_2)^2+(E_1-E_2)^2\right)(x)+C\|b_1-b_2\|^2_{C[0,1]}\\&\leq C\left(\frac{(\rho_1-\rho_2)(x)}{x^{\frac{1}{2}}}\right)^2+C\left((n_1-n_2)^2+(E_1-E_2)^2\right)(x)+C\|b_1-b_2\|^2_{C[0,1]}\\&\leq C\left((n_1-n_2)^2+(E_1-E_2)^2\right)(x)+C\|b_1-b_2\|^2_{C[0,1]},\quad x\in[0,\delta_0).\end{aligned} \tag{3.43}$$

Applying Gronwall's inequality to (3.43), bearing in mind $n_1(0)=n_2(0)=\sigma_0$ and (3.32), we obtain

$$\begin{aligned}\left((n_1-n_2)^2+(E_1-E_2)^2\right)(x)&\leq C\left[(E_1-E_2)^2(0)+\int_0^x\|b_1-b_2\|^2_{C[0,1]}dy\right]\\&\leq C\|b_1-b_2\|^2_{C[0,1]},\quad x\in[0,\delta_0).\end{aligned} \tag{3.44}$$

Therefore,

$$|n_1 - n_2|(x) + |E_1 - E_2|(x) \le C\|b_1 - b_2\|_{C[0,1]}, \quad x \in [0, \delta_0). \tag{3.45}$$

Further, (3.40) and (3.42), together with the estimates (3.27) and (3.45), yield

$$|(n_1 - n_2)_x|(x) + |(E_1 - E_2)_x|(x) \le C\|b_1 - b_2\|_{C[0,1]}, \quad x \in [0, \delta_0). \tag{3.46}$$

Finally, combining (3.27), (3.38), (3.45) and (3.46) leads to the local weighted structural stability estimate (3.23), which completes the proof. □

Next, we are able to establish the local structural stability estimate on the middle non-singular domain. The proof is based on the continuation argument.

Lemma 3.6 *(Local structural stability estimate on the middle domain). Let* $b_1, b_2 \in C[0,1]$ *and* $b_1(x) \ge b_2(x)$ *on* $[0,1]$. *Then there exists a constant* $C > 0$ *independent of* $\|b_1 - b_2\|_{C[0,1]}$ *such that*

$$\|\rho_1 - \rho_2\|_{C^1[\delta_1, 1-\delta_1]} + \|n_1 - n_2\|_{C^1[\delta_1, 1-\delta_1]} + \|E_1 - E_2\|_{C^1[\delta_1, 1-\delta_1]} \le C\|b_1 - b_2\|_{C[0,1]}, \tag{3.47}$$

where $\delta_1 \in (0, \delta_0)$ *and* δ_0 *is determined by Lemma 3.5.*

Proof. Taking the difference of $(3.34)|_{i=1}$ and $(3.34)|_{i=2}$ and using the mean-value theorem yield

$$\begin{aligned}(\rho_1 - \rho_2)_x =& \frac{(\rho_1)^3 E_1}{(\rho_1)^2 - J^2} - \frac{(\rho_2)^3 E_2}{(\rho_2)^2 - J^2} \\ =& E_1\left(\frac{(\rho_1)^3}{(\rho_1)^2 - J^2} - \frac{(\rho_2)^3}{(\rho_2)^2 - J^2}\right) + \frac{(\rho_2)^3}{(\rho_2)^2 - J^2}(E_1 - E_2) \\ =& E_1 f'(\eta)(\rho_1 - \rho_2) + f(\rho_2)(E_1 - E_2), \quad \exists\, \eta \in (\rho_2, \rho_1),\end{aligned} \tag{3.48}$$

where $f(s)$ is defined in (3.41). Multiplying (3.48) by $(\rho_1 - \rho_2)$, (3.40) by $(n_1 - n_2)$ and (3.42) by $(E_1 - E_2)$, using Cauchy's inequality and summing the resulting estimates, we deduce

$$\begin{aligned}&\left((\rho_1 - \rho_2)^2 + (n_1 - n_2)^2 + (E_1 - E_2)^2\right)_x (x) \\ &\quad \le C\left((\rho_1 - \rho_2)^2 + (n_1 - n_2)^2 + (E_1 - E_2)^2\right)(x) + \|b_1 - b_2\|^2_{C[0,1]}, \quad x \in [\delta_1, 1-\delta_1].\end{aligned} \tag{3.49}$$

Applying Gronwall's inequality to (3.49), we have

$$\begin{aligned}&\left((\rho_1 - \rho_2)^2 + (n_1 - n_2)^2 + (E_1 - E_2)^2\right)(x) \\ &\quad \le C\left[\left((\rho_1 - \rho_2)^2 + (n_1 - n_2)^2 + (E_1 - E_2)^2\right)(\delta_1) + \|b_1 - b_2\|^2_{C[0,1]}\right], \quad x \in [\delta_1, 1-\delta_1].\end{aligned} \tag{3.50}$$

Bearing in mind $\delta_1 < \delta_0$, Lemma 3.5 gives

$$\begin{aligned}&\Big((\rho_1-\rho_2)^2+(n_1-n_2)^2+(E_1-E_2)^2\Big)(\delta_1)\\&\le\left(\frac{(\rho_1-\rho_2)(\delta_1)}{(\delta_1)^{\frac12}}\right)^2+(n_1-n_2)^2(\delta_1)+(E_1-E_2)^2(\delta_1)\\&\le\|b_1-b_2\|^2_{C[0,1]},\end{aligned}\tag{3.51}$$

which together with (3.50) yields

$$|\rho_1-\rho_2|(x)+|n_1-n_2|(x)+|E_1-E_2|(x)\le C\|b_1-b_2\|_{C[0,1]},\quad x\in[\delta_1,1-\delta_1].\tag{3.52}$$

Moreover, by (3.40), (3.42), (3.48) and (3.52), we obtain

$$|(\rho_1-\rho_2)_x|(x)+|(n_1-n_2)_x|(x)+|(E_1-E_2)_x|(x)\le C\|b_1-b_2\|_{C[0,1]},\quad x\in[\delta_1,1-\delta_1].\tag{3.53}$$

Combining (3.52) and (3.53) implies the local structural stability estimate (3.47) on the middle non-singular domain $[\delta_1,1-\delta_1]$. □

Finally, we establish the local weighted structural stability estimate near the right endpoint $x=1$.

Lemma 3.7 *(Local weighted structural stability estimate near $x=1$). Let $b_1,b_2\in C[0,1]$ and $b_1(x)\ge b_2(x)$ on $[0,1]$. Then there exists a constant $C>0$ independent of $\|b_1-b_2\|_{C[0,1]}$ such that*

$$\begin{aligned}&\|(1-x)^{-\frac12}(\rho_1-\rho_2)\|_{C(1-\delta_0,1]}+\|(1-x)^{\frac12}(\rho_1-\rho_2)_x\|_{C(1-\delta_0,1]}\\&\quad+\|n_1-n_2\|_{C^1(1-\delta_0,1]}+\|E_1-E_2\|_{C^1(1-\delta_0,1]}\le C\|b_1-b_2\|_{C[0,1]},\end{aligned}\tag{3.54}$$

where δ_0 is determined by Lemma 3.5.

Proof. We first claim that there exists a constant $M_1>0$ independent of $\|b_1-b_2\|_{C[0,1]}$ such that

$$\frac{(\rho_1-\rho_2)(x)}{(1-x)^{\frac12}}\le M_1\|b_1-b_2\|_{C[0,1]},\quad x\in(1-\delta_0,1].\tag{3.55}$$

Otherwise, by a reasoning similar to that in the proof of Lemma 3.5, for any $\widetilde{M}>0$, there exist $x_l\in(1-\frac1l,1]$ with $l=3,4,5,\cdots$, such that

$$\varliminf_{x_l\to1^-}\frac{(\rho_1-\rho_2)(x_l)}{(1-x_l)^{\frac12}}\ge\widetilde{M}\|b_1-b_2\|_{C[0,1]}.\tag{3.56}$$

In fact, by $(3.1)_3$, (3.24), (3.26) and Lemma 3.4, we obtain

$$
\begin{aligned}
\lim_{x\to 1^-} \frac{(\rho_1-\rho_2)(x)}{(1-x)^{\frac{1}{2}}} &= J\left(\sqrt{-E_1(1)}-\sqrt{-E_2(1)}\right) \\
&= \mathcal{C}_0\left(-E_1(1)+E_2(1)\right) \\
&\le \mathcal{C}_0\Big(\,(E_1(0)-E_1(1))-(E_2(0)-E_2(1))\,\Big) \\
&= \mathcal{C}_0\left(\int_0^1 (b_1+n_1-\rho_1)dx-\int_0^1 (b_2+n_2-\rho_2)dx\right) \\
&= \mathcal{C}_0\left(\int_0^1 (b_1-b_2)dx+\int_0^1 (n_1-n_2)dx-\int_0^1 (\rho_1-\rho_2)dx\right) \\
&\le \mathcal{C}_0\int_0^1 (b_1-b_2)dx \\
&\le \widetilde{\mathcal{C}_0}\|b_1-b_2\|_{C[0,1]},
\end{aligned}
\tag{3.57}
$$

where $\mathcal{C}_0=\frac{J}{\sqrt{-E_1(1)}+\sqrt{-E_2(1)}}>0$ and $\widetilde{\mathcal{C}_0}>0$ is a constant independent of $\|b_1-b_2\|_{C[0,1]}$. Therefore, taking $\widetilde{M}=2\widetilde{\mathcal{C}_0}$ yields a contradiction to (3.56), which implies (3.55). Analogous to (3.32), it is also worth mentioning that (3.57) shows

$$
E_2(1)-E_1(1)\le C\|b_1-b_2\|_{C[0,1]}, \tag{3.58}
$$

that is, the difference between the values of electric field at $x=1$ can be controlled by the perturbation of doping profiles. Moreover, by arguing similarly as in (3.33), for $x\in(1-\delta_0,1]$, we deduce

$$
(n_2-n_1)(x)\le C(w(n_2)-w(n_1))=C(w(\rho_1)-w(\rho_2))\le \frac{C(\rho_1-\rho_2)}{(1-x)^{\frac{1}{2}}}\le C\|b_1-b_2\|_{C[0,1]}. \tag{3.59}
$$

Now, we are ready to prove (3.54). Multiplying (3.34) by $(1-x)^{\frac{1}{2}}$ and taking the difference of the resulting equations for $i=1,2$, we have

$$
\begin{aligned}
(1-x)^{\frac{1}{2}}(\rho_1-\rho_2)_x &= \frac{(\rho_1)^3E_1}{\rho_1+J}\frac{(1-x)^{\frac{1}{2}}}{\rho_1-J}-\frac{(\rho_2)^3E_2}{\rho_2+J}\frac{(1-x)^{\frac{1}{2}}}{\rho_2-J} \\
&= \frac{(\rho_1)^3E_1}{\rho_1+J}\left(\frac{(1-x)^{\frac{1}{2}}}{\rho_1-J}-\frac{(1-x)^{\frac{1}{2}}}{\rho_2-J}\right)+\left(\frac{(\rho_1)^3E_1}{\rho_1+J}-\frac{(\rho_2)^3E_2}{\rho_2+J}\right)\frac{(1-x)^{\frac{1}{2}}}{\rho_2-J} \\
&=: K_1+K_2.
\end{aligned}
\tag{3.60}
$$

Using (3.12) and (3.55) yields

$$
|K_1|=\left|\frac{(\rho_1)^3E_1}{\rho_1+J}\frac{(1-x)^{\frac{1}{2}}}{\rho_1-J}\frac{(1-x)^{\frac{1}{2}}}{\rho_2-J}\frac{\rho_2-\rho_1}{(1-x)^{\frac{1}{2}}}\right|\le C\left(\frac{\rho_1-\rho_2}{(1-x)^{\frac{1}{2}}}\right)\le C\|b_1-b_2\|_{C[0,1]}. \tag{3.61}
$$

Combining (3.12), (3.17), (3.24), (3.55), (3.58), (3.59) and the mean-value theorem, we get

$$
\begin{aligned}
|K_2| &= \left| \left(\frac{(\rho_1)^3 E_1}{\rho_1 + J} - \frac{(\rho_2)^3 E_2}{\rho_2 + J} \right) \frac{(1-x)^{\frac{1}{2}}}{\rho_2 - J} \right| \\
&\le C \left| \frac{(\rho_1)^3 E_1}{\rho_1 + J} - \frac{(\rho_2)^3 E_2}{\rho_2 + J} \right| \\
&= C \left| E_1 \left(\frac{(\rho_1)^3}{\rho_1 + J} - \frac{(\rho_2)^3}{\rho_2 + J} \right) + \frac{(\rho_2)^3}{\rho_2 + J}(E_1 - E_2) \right| \\
&\le C|\rho_1 - \rho_2|(x) + C \left| (E_1(1) - E_2(1)) - \int_x^1 \big[(\rho_1 - \rho_2) - (n_1 - n_2) - (b_1 - b_2)\big] dy \right| \\
&\le C(\rho_1 - \rho_2)(x) + C(E_2(1) - E_1(1)) + C \int_x^1 (\rho_1 - \rho_2) dy \\
&\quad + C \int_x^1 (n_2 - n_1) dy + C\|b_1 - b_2\|_{C[0,1]} \\
&\le C \left(\frac{(\rho_1 - \rho_2)(x)}{(1-x)^{\frac{1}{2}}} + \frac{(\rho_1 - \rho_2)(\theta)}{(1-\theta)^{\frac{1}{2}}} \right) + C(E_2(1) - E_1(1)) \\
&\quad + C(n_2 - n_1)(\bar{\theta}) + C\|b_1 - b_2\|_{C[0,1]}, \quad \exists\, \theta, \bar{\theta} \in [x, 1], \\
&\le C\|b_1 - b_2\|_{C[0,1]}.
\end{aligned} \tag{3.62}
$$

Inserting (3.61) and (3.62) into (3.60) implies that

$$
(1-x)^{\frac{1}{2}} |(\rho_1 - \rho_2)_x|(x) \le C\|b_1 - b_2\|_{C[0,1]}, \quad x \in (1 - \delta_0, 1]. \tag{3.63}
$$

Analogous to (3.43), we obtain

$$
\begin{aligned}
&\Big((n_1 - n_2)^2 + (E_1 - E_2)^2 \Big)_x (x) \\
&\le C \Big((\rho_1 - \rho_2)^2 + (n_1 - n_2)^2 + (E_1 - E_2)^2 \Big)(x) + C\|b_1 - b_2\|^2_{C[0,1]} \\
&\le C \left(\frac{(\rho_1 - \rho_2)(x)}{(1-x)^{\frac{1}{2}}} \right)^2 + C \Big((n_1 - n_2)^2 + (E_1 - E_2)^2 \Big)(x) + C\|b_1 - b_2\|^2_{C[0,1]} \\
&\le C \Big((n_1 - n_2)^2 + (E_1 - E_2)^2 \Big)(x) + C\|b_1 - b_2\|^2_{C[0,1]}, \quad x \in (1 - \delta_0, 1].
\end{aligned} \tag{3.64}
$$

An application of Gronwall's inequality to (3.64) gives

$$\begin{aligned}&\Big((n_1-n_2)^2+(E_1-E_2)^2\Big)(x)\\&\quad\le C\left[(n_1-n_2)^2(1-\delta_0)+(E_1-E_2)^2(1-\delta_0)+\|b_1-b_2\|^2_{C[0,1]}\right],\quad x\in(1-\delta_0,1].\end{aligned}\tag{3.65}$$

Since $1-\delta_0<1-\delta_1$, by Lemma 3.6, we have

$$(n_1-n_2)^2(1-\delta_0)+(E_1-E_2)^2(1-\delta_0)\le C\|b_1-b_2\|^2_{C[0,1]},\tag{3.66}$$

which together with (3.65) yields

$$|n_1-n_2|(x)+|E_1-E_2|(x)\le C\|b_1-b_2\|_{C[0,1]},\quad x\in(1-\delta_0,1].\tag{3.67}$$

Combining (3.40), (3.42), (3.55) and (3.67) implies that

$$|(n_1-n_2)_x|(x)+|(E_1-E_2)_x|(x)\le C\|b_1-b_2\|_{C[0,1]},\quad x\in(1-\delta_0,1].\tag{3.68}$$

Hence, the local weighted structural stability estimate (3.54) follows from (3.55), (3.63), (3.67) and (3.68). □

Based on Lemmas 3.5-3.7, we are now in position to construct the structural stability of subsonic solution to (3.1) under a monotonicity condition on doping profiles.

Proposition 3.1. *Let $b_1,b_2\in C[0,1]$ and $b_1(x)\ge b_2(x)$ on $[0,1]$. For $i=1,2$, let $(\rho_i,n_i,E_i)(x)$ denote the subsonic solution to (3.1) corresponding to $b_i(x)$. Then there exist two constants $\delta_0\in(0,\frac{1}{2})$ and $C>0$ independent of $\|b_1-b_2\|_{C[0,1]}$ such that*

$$\begin{aligned}&\|\rho_1-\rho_2\|_{C[0,1]}+\|x^{\frac12}(\rho_1-\rho_2)_x\|_{C[0,\delta_0)}+\|(\rho_1-\rho_2)_x\|_{C[\delta_0,1-\delta_0]}\\&\quad+\|(1-x)^{\frac12}(\rho_1-\rho_2)_x\|_{C(1-\delta_0,1]}\\&\quad+\|n_1-n_2\|_{C^1[0,1]}+\|E_1-E_2\|_{C^1[0,1]}\le C\|b_1-b_2\|_{C[0,1]}.\end{aligned}\tag{3.69}$$

Proof. Combining (3.23), (3.47) and (3.54), we obtain (3.69), as desired. □

Now, we are ready to prove Theorem 1.2, which removes the monotonicity restriction $b_1(x)\ge b_2(x)$ on $[0,1]$ in Proposition 3.1 by the squeezing skill.

Proof of Theorem 1.2. Let $(\varrho,\mathcal{N},\mathcal{E})(x)$ denote the subsonic solution to (3.1) corresponding to the doping profile $B(x):=\max\{b_1(x),b_2(x)\}$. Then, it follows from Proposition 3.1 that

$$
\begin{aligned}
&\|\rho_1-\rho_2\|_{C[0,1]}+\|x^{\frac{1}{2}}(\rho_1-\rho_2)_x\|_{C[0,\delta_0)}+\|(\rho_1-\rho_2)_x\|_{C[\delta_0,1-\delta_0]}\\
&\quad+\|(1-x)^{\frac{1}{2}}(\rho_1-\rho_2)_x\|_{C(1-\delta_0,1]}+\|n_1-n_2\|_{C^1[0,1]}+\|E_1-E_2\|_{C^1[0,1]}\\
&\le\|\rho_1-\varrho\|_{C[0,1]}+\|x^{\frac{1}{2}}(\rho_1-\varrho)_x\|_{C[0,\delta_0)}+\|(\rho_1-\varrho)_x\|_{C[\delta_0,1-\delta_0]}\\
&\quad+\|(1-x)^{\frac{1}{2}}(\rho_1-\varrho)_x\|_{C(1-\delta_0,1]}+\|n_1-\mathcal{N}\|_{C^1[0,1]}+\|E_1-\mathcal{E}\|_{C^1[0,1]}\\
&\quad+\|\varrho-\rho_2\|_{C[0,1]}+\|x^{\frac{1}{2}}(\varrho-\rho_2)_x\|_{C[0,\delta_0)}+\|(\varrho-\rho_2)_x\|_{C[\delta_0,1-\delta_0]}\\
&\quad+\|(1-x)^{\frac{1}{2}}(\varrho-\rho_2)_x\|_{C(1-\delta_0,1]}+\|\mathcal{N}-n_2\|_{C^1[0,1]}+\|\mathcal{E}-E_2\|_{C^1[0,1]}\\
&\le C\|B-b_1\|_{C[0,1]}+C\|B-b_2\|_{C[0,1]}\\
&\le C\|b_1-b_2\|_{C[0,1]}.
\end{aligned}
\tag{3.70}
$$

This completes the proof of Theorem 1.2. □

4. Numerical simulations

This section presents some numerical simulations with respect to different types of doping profiles. Owing to the degeneracy of electrons at the boundary, the numerical simulations to (1.3)-(1.4) cannot be carried out directly. An alternative is to consider the following subsonic-current-approximation system:

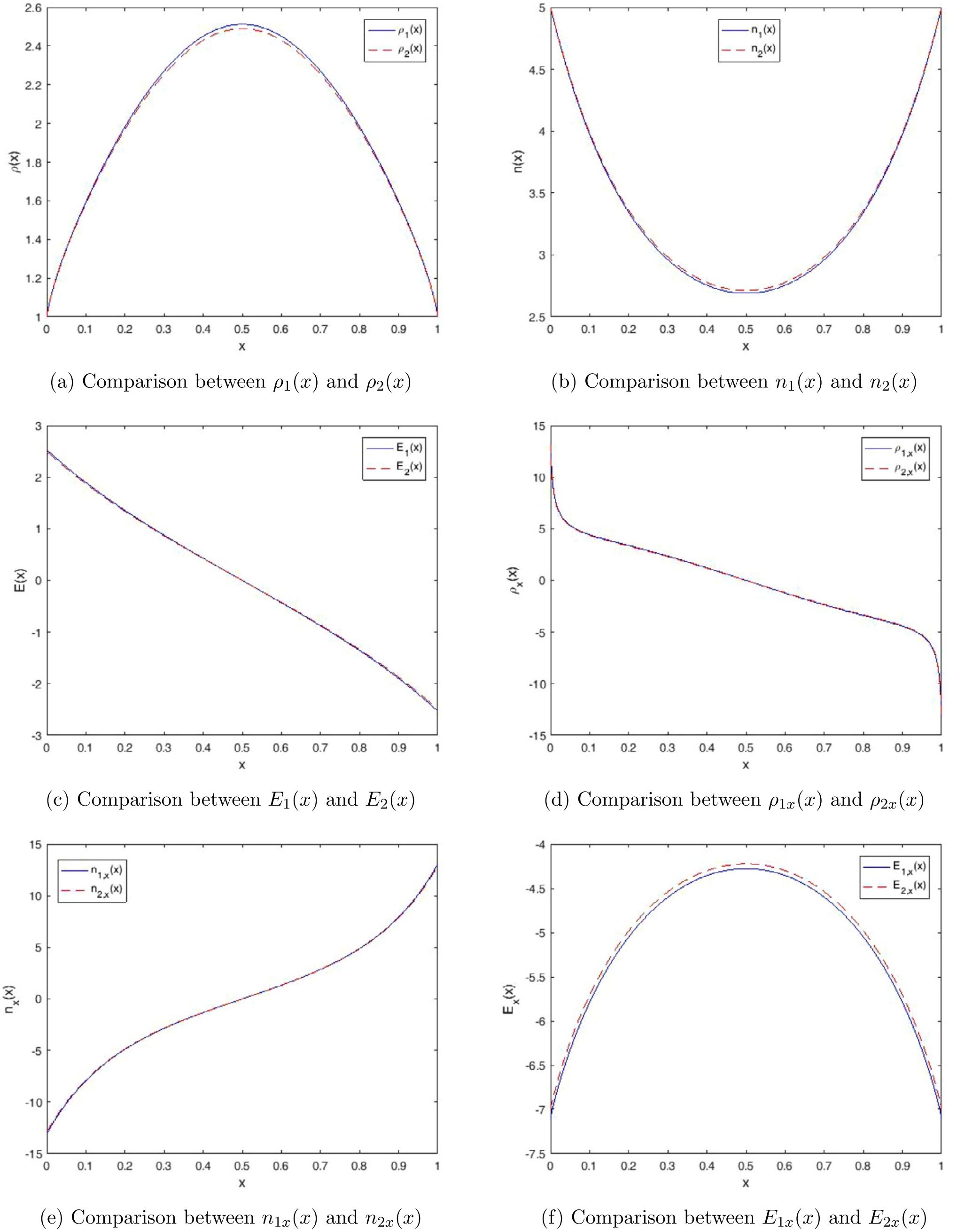
$$
\begin{cases}
\rho_x=\dfrac{\rho E}{1-j^2/\rho^2},\\
n_x=-\dfrac{nE}{1-j^2/n^2}, & x\in(0,1),\\
E_x=\rho-n-b(x),\\
\rho(0)=\rho(1)=J,\quad n(0)=\sigma_0>J,
\end{cases}
\tag{4.1}
$$

where $0<j<J$. For $i=1,2$, let $(\rho_i,n_i,E_i)(x)$ denote the numerical solution to (4.1) corresponding to $b_i(x)$. In order to facilitate numerical calculations, we set $j=0.9$, $J=1$ and $\sigma_0=5$. The following three different types of doping profiles will be considered separately:

(i) Subsonic doping profiles: $b_1(x)=3.1+\sin(\pi x)$ and $b_2(x)=3+\sin(\pi x)$;
(ii) Supersonic doping profiles: $b_1(x)=0.6+0.3\sin(\pi x)$ and $b_2(x)=0.5+0.3\sin(\pi x)$;
(iii) Transonic doping profiles: $b_1(x)=1.1+0.5\sin(\pi x)$ and $b_2(x)=1+0.5\sin(\pi x)$.

The numerical simulations are performed in Figs. 1-3.

Let us take the type (i) as an example; see Fig. 1 for the corresponding numerical simulations. Fig. 1(a) and (b) indicate that the comparison principle in Lemma 3.4 holds. From Fig. 1(c), it can be seen that $E_1(1)\le E_2(1)<0<E_2(0)\le E_1(0)$, which is consistent with (3.24). From Fig. 1(a), (b), (c), (e) and (f), we observe that $\|\rho_1-\rho_2\|_{C[0,1]}+\|n_1-n_2\|_{C^1[0,1]}+\|E_1-E_2\|_{C^1[0,1]}$ can be controlled by $\|b_1-b_2\|_{C[0,1]}$. Further, Fig. 1(d) suggests that the first order derivative of the electron density blows up near $x=0$ and $x=1$ (i.e., (3.14)), which coincides



(a) Comparison between $\rho_1(x)$ and $\rho_2(x)$

(b) Comparison between $n_1(x)$ and $n_2(x)$

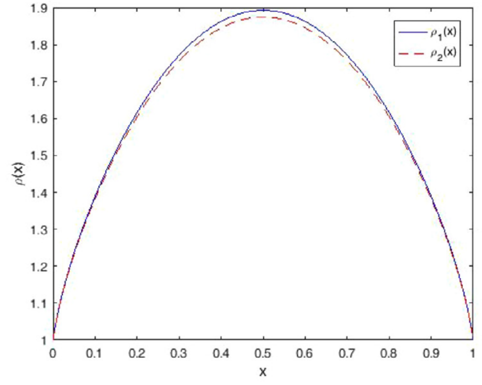
(c) Comparison between $E_1(x)$ and $E_2(x)$

(d) Comparison between $\rho_{1x}(x)$ and $\rho_{2x}(x)$

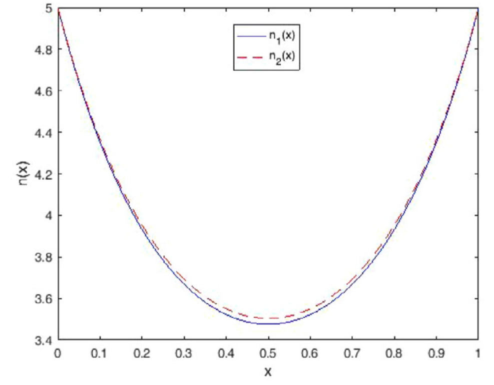
(e) Comparison between $n_{1x}(x)$ and $n_{2x}(x)$

(f) Comparison between $E_{1x}(x)$ and $E_{2x}(x)$

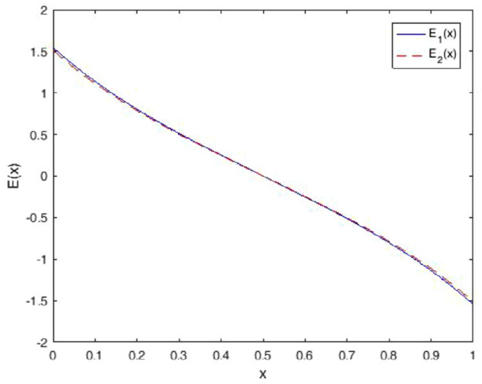
Fig. 1. Structural stability between $(\rho_1, n_1, E_1)(x)$ and $(\rho_2, n_2, E_2)(x)$ corresponding to subsonic doping profiles $b_1(x) = 3.1 + \sin(\pi x)$ and $b_2(x) = 3 + \sin(\pi x)$.



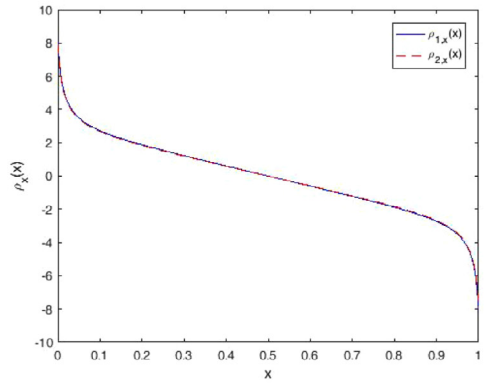
(a) Comparison between $\rho_1(x)$ and $\rho_2(x)$



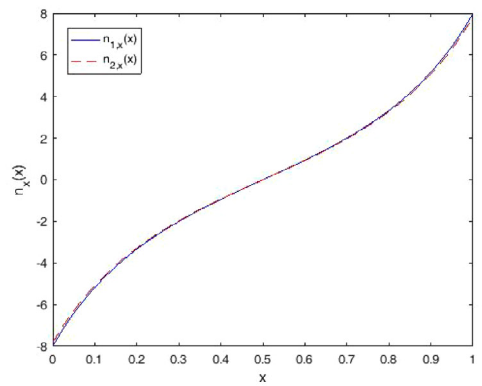
(b) Comparison between $n_1(x)$ and $n_2(x)$



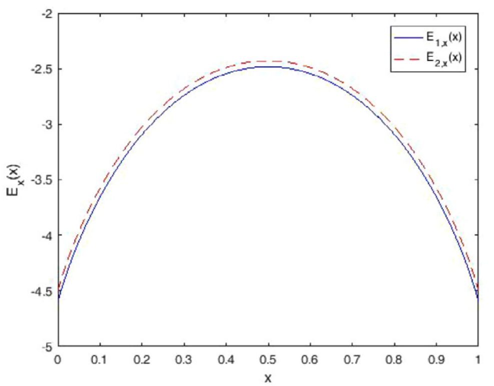
(c) Comparison between $E_1(x)$ and $E_2(x)$



(d) Comparison between $\rho_{1x}(x)$ and $\rho_{2x}(x)$

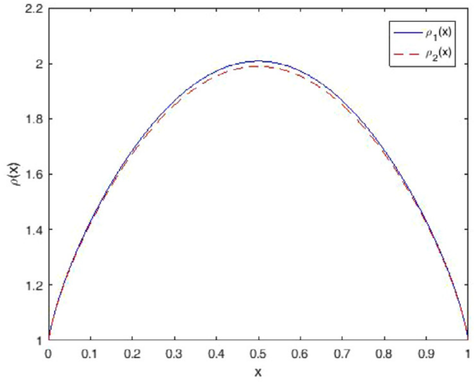


(e) Comparison between $n_{1x}(x)$ and $n_{2x}(x)$

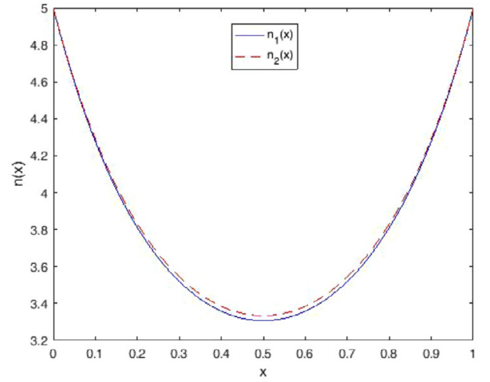


(f) Comparison between $E_{1x}(x)$ and $E_{2x}(x)$

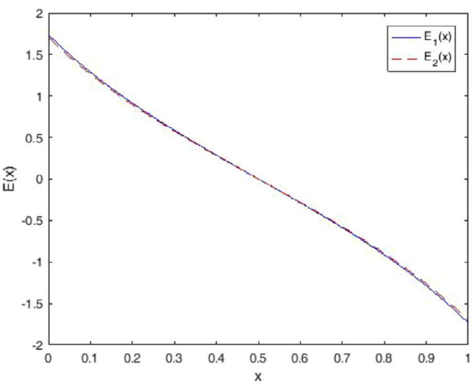
Fig. 2. Structural stability between $(\rho_1, n_1, E_1)(x)$ and $(\rho_2, n_2, E_2)(x)$ corresponding to supersonic doping profiles $b_1(x) = 0.6 + 0.3\sin(\pi x)$ and $b_2(x) = 0.5 + 0.3\sin(\pi x)$.



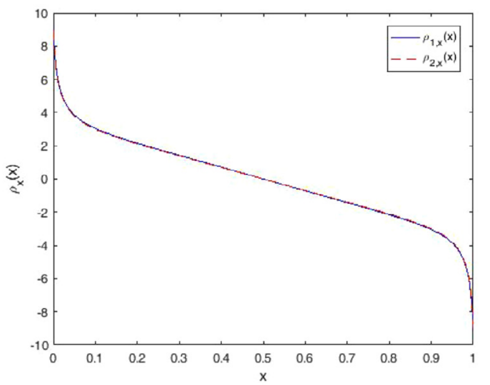
(a) Comparison between $\rho_1(x)$ and $\rho_2(x)$



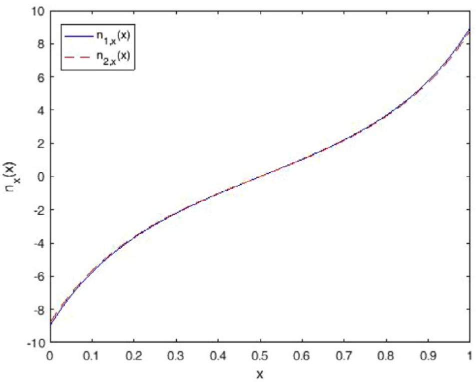
(b) Comparison between $n_1(x)$ and $n_2(x)$



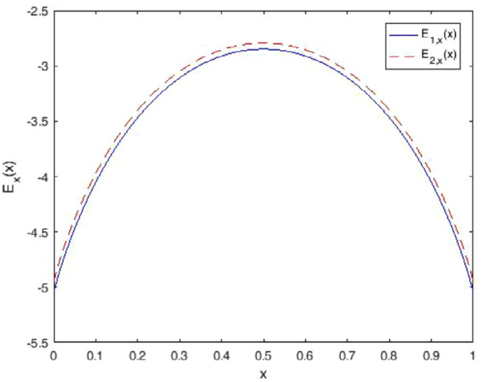
(c) Comparison between $E_1(x)$ and $E_2(x)$



(d) Comparison between $\rho_{1x}(x)$ and $\rho_{2x}(x)$



(e) Comparison between $n_{1x}(x)$ and $n_{2x}(x)$



(f) Comparison between $E_{1x}(x)$ and $E_{2x}(x)$

Fig. 3. Structural stability between $(\rho_1, n_1, E_1)(x)$ and $(\rho_2, n_2, E_2)(x)$ corresponding to transonic doping profiles $b_1(x) = 1.1 + 0.5\sin(\pi x)$ and $b_2(x) = 1 + 0.5\sin(\pi x)$.

with Lemmas 3.1-3.3. Hence, it is quite reasonable to introduce the $x^{\frac{1}{2}}$-weight to control the singularity near $x = 0$ and the $(1-x)^{\frac{1}{2}}$-weight to control the singularity near $x = 1$.

Consequently, the numerical simulations performed sufficiently support our theoretical results, confirming the structural stability of the subsonic solution to (1.3)-(1.4). Furthermore, as can be seen from Figs. 1-3, the structural stability for the bipolar model can be established regardless of the type of doping profile, which is different from the unipolar case.

Data availability

No data was used for the research described in the article.

Acknowledgments

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