MATH 203/2 FALL 2006
ASSIGNMENT 8 (WEEK 9) SOLUTIONS

Section 3.11

6. $f(x)=\ln x \Rightarrow f'(x)=1/x$, so $f(1)=0$ and $f'(1)=1$. Thus, $L(x)=f(1)+f'(1)(x-1)=0+1(x-1)=x-1$.

8. $f(x)=\sqrt[3]{x}=x^{1/3} \Rightarrow f'(x)=\frac{1}{3}x^{-2/3}$, so $f(-8)=-2$ and $f'(-8)=\frac{1}{12}$. Thus,

20. $y=(1+2r)^{-4} \Rightarrow dy=-4(1+2r)^{-5}\cdot 2\,dr=-8(1+2r)^{-5}\,dr$

24. **(a)** $y=1/(x+1) \Rightarrow dy=-\frac{1}{(x+1)^2}\,dx$

(b) When $x=1$ and $dx=-0.01$, $dy=-\frac{1}{2^2}(-0.01)=\frac{1}{4}\cdot\frac{1}{100}=\frac{1}{400}=0.0025$.

36. $y=f(x)=\ln x \Rightarrow dy=\frac{1}{x}\,dx$. When $x=1$ and $dx=0.07$, $dy=\frac{1}{1}(0.07)=0.07$, so $\ln 1.07=f(1.07)\approx f(1)+dy=0+0.07=0.07$.

42. **(a)** $A=\pi r^2 \Rightarrow dA=2\pi r\,dr$. When $r=24$ and $dr=0.2$, $dA=2\pi(24)(0.2)=9.6\pi$, so the maximum possible error in the calculated area of the disk is about $9.6\pi\approx 30\text{ cm}^2$.

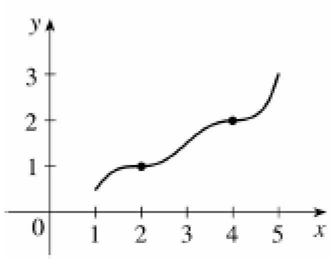
(b) Relative error $=\frac{\Delta A}{A}\approx\frac{dA}{A}=\frac{2\pi r\,dr}{\pi r^2}=\frac{2dr}{r}=\frac{2(0.2)}{24}=\frac{0.2}{12}=\frac{1}{60}=0.01\overline{6}$.

Percentage error = relative error $\times 100\% = 0.01\overline{6}\times 100\% = 1.\overline{6}\%$.

Section 4.1

6. Absolute maximum value is $f(8)=5$; absolute minimum value is $f(2)=0$; local maximum values are $f(1)=2$, $f(4)=4$, and $f(6)=3$; local minimum values are $f(2)=0$, $f(5)=2$, and $f(7)=1$.

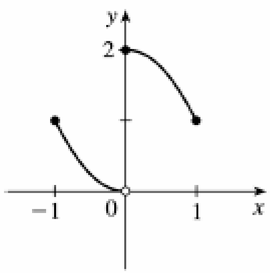
10. f has no local maximum or minimum, but 2 and 4 are critical numbers



30. $f(x)=\begin{cases} x^2 & \text{if } -1\leq x<0 \\ 2-x^2 & \text{if } 0\leq x\leq 1 \end{cases}$

Absolute and local maximum $f(0)=2$.

No absolute or local minimum.



42. $G(x)=\sqrt[3]{x^2-x} \Rightarrow G'(x)=\frac{1}{3}\left(x^2-x\right)^{-2/3}(2x-1)$. $G'(x)$ does not exist when $x^2-x=0$, that is, when $x=0$ or 1. $G'(x)=0 \Leftrightarrow 2x-1=0 \Leftrightarrow x=\frac{1}{2}$. So the critical numbers are $x=0$, $\frac{1}{2}$, 1.

46. $f(x)=xe^{2x} \Rightarrow f'(x)=x(2e^{2x})+e^{2x}=e^{2x}(2x+1)$. Since e^{2x} is never 0 , we have $f'(x)=0$ only when $2x+1=0 \Leftrightarrow x=-\frac{1}{2}$. So $-\frac{1}{2}$ is the only critical number.

62. $f(x)=e^{-x}-e^{-2x}$, $[0,1]$. $f'(x)=e^{-x}(-1)-e^{-2x}(-2)=\frac{2}{e^{2x}}-\frac{1}{e^{x}}=\frac{2-e^{x}}{e^{2x}}=0 \Leftrightarrow e^{x}=2 \Leftrightarrow x=\ln 2\approx 0.69$. $f(0)=0$, $f(\ln 2)=e^{-\ln 2}-e^{-2\ln 2}=\left(e^{\ln 2}\right)^{-1}-\left(e^{\ln 2}\right)^{-2}=2^{-1}-2^{-2}=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$, $f(1)=e^{-1}-e^{-2}\approx 0.233$. So $f(\ln 2)=\frac{1}{4}$ is the absolute maximum value and $f(0)=0$ is the absolute minimum value.