MATH 203/2 FALL 2006
ASSIGNMENT 7 (WEEK 8) SOLUTIONS

Section 3.7

4. a must be the jerk since none of the graphs are 0 at its high and low points. a is 0 where b has a maximum, so $b' = a$. b is 0 where c has a maximum, so $c' = b$. We conclude that d is the position function, c is the velocity, b is the acceleration, and a is the jerk.

12.

$$H(s) = a\sqrt{s} + \frac{b}{\sqrt{s}} = as^{1/2} + bs^{-1/2} \Rightarrow$$

$$H'(s) = a \cdot \frac{1}{2} s^{-1/2} + b\left(-\frac{1}{2} s^{-3/2}\right) = \frac{1}{2} as^{-1/2} - \frac{1}{2} bs^{-3/2} \Rightarrow$$

$$H''(s) = \frac{1}{2} a\left(-\frac{1}{2} s^{-3/2}\right) - \frac{1}{2} b\left(-\frac{3}{2} s^{-5/2}\right) = -\frac{1}{4} as^{-3/2} + \frac{3}{4} bs^{-5/2}$$

14. $y = xe^{cx} \Rightarrow y' = x \cdot e^{cx} \cdot c + e^{cx} \cdot 1 = e^{cx}(cx+1) \Rightarrow y'' = e^{cx}(c) + (cx+1)e^{cx} \cdot c = ce^{cx}(1+cx+1) = ce^{cx}(cx+2)$

20. $h(x) = \tan^{-1}(x^2) \Rightarrow h'(x) = \dfrac{1}{1+(x^2)^2} \cdot 2x = \dfrac{2x}{1+x^4} \Rightarrow h''(x) = \dfrac{(1+x^4)(2)-(2x)(4x^3)}{(1+x^4)^2} = \dfrac{2-6x^4}{(1+x^4)^2}$

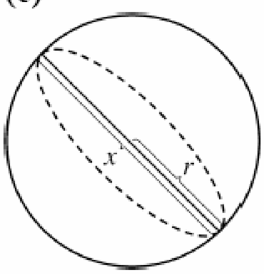
34. $f(x) = \dfrac{1}{5x-1} = (5x-1)^{-1} \Rightarrow f'(x) = -1(5x-1)^{-2} \cdot 5 \Rightarrow f''(x) = (-1)(-2)(5x-1)^{-3} \cdot 5^2 \Rightarrow$
$f'''(x) = (-1)(-2)(-3)(5x-1)^{-4} \cdot 5^3 \Rightarrow \cdots \Rightarrow f^{(n)}(x) = (-1)^n n!\, 5^n (5x-1)^{-(n+1)}$

36. $f(x) = \sqrt{x} = x^{1/2} \Rightarrow f'(x) = \dfrac{1}{2} x^{-1/2} \Rightarrow$

Section 3.10

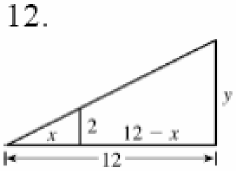
6. $y=\sqrt{1+x^3} \Rightarrow \frac{dy}{dt}=\frac{dy}{dx}\frac{dx}{dt}=\frac{1}{2}(1+x^3)^{-1/2}(3x^2)\frac{dx}{dt}=\frac{3x^2}{2\sqrt{1+x^3}}\frac{dx}{dt}$. With $\frac{dy}{dt}=4$ when $x=2$ and $y=3$, we have $4=\frac{3(4)}{2(3)}\frac{dx}{dt} \Rightarrow \frac{dx}{dt}=2$ cm / s.

8. **(a)** Given: the rate of decrease of the surface area is 1 cm^2/min. If we let t be time (in minutes) and S be the surface area (in cm^2), then we are given that $dS/dt=-1$ cm^2/ s.
(b) Unknown: the rate of decrease of the diameter when the diameter is 10 cm. If we let x be the diameter, then we want to find dx/dt when $x=10$ cm.
(c)



(d) If the radius is r and the diameter $x=2r$, then $r=\frac{1}{2}x$ and $S=4\pi r^2=4\pi\left(\frac{1}{2}x\right)^2=\pi x^2 \Rightarrow$
$\frac{dS}{dt}=\frac{dS}{dx}\frac{dx}{dt}=2\pi x\frac{dx}{dt}$.
(e) $-1=\frac{dS}{dt}=2\pi x\frac{dx}{dt} \Rightarrow \frac{dx}{dt}=-\frac{1}{2\pi x}$. When $x=10$, $\frac{dx}{dt}=-\frac{1}{20\pi}$, So the rate of decrease is $\frac{1}{20\pi}$ cm/min.

12.



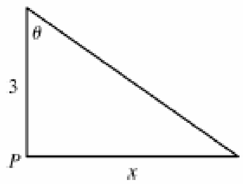
We are given that $\frac{dx}{dt}=1.6$ m/s. By similar triangles, $\frac{y}{12}=\frac{2}{x} \Rightarrow y=\frac{24}{x} \Rightarrow \frac{dy}{dt}=-\frac{24}{x^2}\frac{dx}{dt}=-\frac{24}{x^2}(1.6)$.

When $x=8$, $\frac{dy}{dt}=-\frac{24(1.6)}{64}=-0.6$ m/s, so the shadow is decreasing at a rate of 0.6 m/s.

18. Let D denote the distance from the origin (0,0) to the point on the curve $y=\sqrt{x}$.

$D=\sqrt{(x-0)^2+(y-0)^2}=\sqrt{x^2+(\sqrt{x})^2}=\sqrt{x^2+x} \Rightarrow \frac{dD}{dt}=\frac{1}{2}(x^2+x)^{-1/2}(2x+1)\frac{dx}{dt}=\frac{2x+1}{2\sqrt{x^2+x}}\frac{dx}{dt}$. With $\frac{dx}{dt}=3$ when $x=4$, $\frac{dD}{dt}=\frac{9}{2\sqrt{20}}(3)=\frac{27}{4\sqrt{5}}\approx 3.02$ cm / s.

34.



We are given that $\frac{d\theta}{dt}=4(2\pi)=8\pi$ rad/min. $x=3\tan\theta \Rightarrow \frac{dx}{dt}=3\sec^2\theta\frac{d\theta}{dt}$. When $x=1$, $\tan\theta=\frac{1}{3}$, so $\sec^2\theta=1+\left(\frac{1}{3}\right)^2=\frac{10}{9}$ and $\frac{dx}{dt}=3\left(\frac{10}{9}\right)(8\pi)=\frac{80\pi}{3}\approx 83.8$ km/min.