MATH 203/2 FALL 2006
ASSIGNMENT 6 SOLUTIONS

Section 3.5

6. Let $u=g(x)=e^x$ and $y=f(u)=\sin u$. Then $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}=(\cos u)(e^x)=e^x\cos e^x$.

12. $f(t)=\sqrt[3]{1+\tan t}=(1+\tan t)^{1/3}\Rightarrow f'(t)=\frac{1}{3}(1+\tan t)^{-2/3}\sec^2 t=\frac{\sec^2 t}{3\sqrt[3]{(1+\tan t)^2}}$

20. $y=(x^2+1)(x^2+2)^{1/3}\Rightarrow y'=2x(x^2+2)^{1/3}+(x^2+1)\left(\frac{1}{3}\right)(x^2+2)^{-2/3}(2x)=2x(x^2+2)^{1/3}\left[1+\frac{x^2+1}{3(x^2+2)}\right]$

24. Using Formula 5 and the Chain Rule, $y=10^{1-x^2}\Rightarrow y'=10^{1-x^2}(\ln 10)\cdot\frac{d}{dx}(1-x^2)=-2x(\ln 10)10^{1-x^2}$.

38. $y=\sin(\sin(\sin x))\Rightarrow y'=\cos(\sin(\sin x))\frac{d}{dx}(\sin(\sin x))=\cos(\sin(\sin x))\cdot\cos(\sin x)\cdot\cos x$

40. $y=\sqrt{x+\sqrt{x+\sqrt{x}}}\Rightarrow y'=\frac{1}{2}\left(x+\sqrt{x+\sqrt{x}}\right)^{-1/2}\left[1+\frac{1}{2}(x+\sqrt{x})^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)\right]$

Section 3.6

2. **(a)** $\frac{d}{dx}(4x^2+9y^2)=\frac{d}{dx}(36)\Rightarrow 8x+18y\cdot y'=0\Rightarrow y'=-\frac{8x}{18y}=-\frac{4x}{9y}$

12. $\frac{d}{dx}(1+x)=\frac{d}{dx}[\sin(xy^2)]\Rightarrow 1=[\cos(xy^2)](x\cdot 2yy'+y^2\cdot 1)\Rightarrow 1=2xy\cos(xy^2)y'+y^2\cos(xy^2)\Rightarrow$

$1-y^2\cos(xy^2)=2xy\cos(xy^2)y'\Rightarrow y'=\frac{1-y^2\cos(xy^2)}{2xy\cos(xy^2)}$

20. $\sin x+\cos y=\sin x\cos y \Rightarrow \cos x-\sin y\cdot y'=\sin x(-\sin y\cdot y')+\cos y\cos x \Rightarrow$
$(\sin x\sin y-\sin y)y'=\cos x\cos y-\cos x \Rightarrow y'=\dfrac{\cos x(\cos y-1)}{\sin y(\sin x-1)}$

42. $y=\sqrt{\tan^{-1}x}=(\tan^{-1}x)^{1/2} \Rightarrow$
$y'=\frac{1}{2}(\tan^{-1}x)^{-1/2}\cdot\frac{d}{dx}(\tan^{-1}x)=\dfrac{1}{2\sqrt{\tan^{-1}x}}\cdot\dfrac{1}{1+x^2}=\dfrac{1}{2\sqrt{\tan^{-1}x}\,(1+x^2)}$

44. $h(x)=\sqrt{1-x^2}\,\arcsin x \Rightarrow h'(x)=\sqrt{1-x^2}\cdot\dfrac{1}{\sqrt{1-x^2}}+\arcsin x\left[\frac{1}{2}(1-x^2)^{-1/2}(-2x)\right]=1-\dfrac{x\arcsin x}{\sqrt{1-x^2}}$

46. $y=\tan^{-1}\left(x-\sqrt{x^2+1}\right) \Rightarrow$
$$y'=\frac{1}{1+\left(x-\sqrt{x^2+1}\right)^2}\left(1-\frac{x}{\sqrt{x^2+1}}\right)=\frac{1}{1+x^2-2x\sqrt{x^2+1}+x^2+1}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}}\right)$$
$$=\frac{\sqrt{x^2+1}-x}{2\left(1+x^2-x\sqrt{x^2+1}\right)\sqrt{x^2+1}}=\frac{\sqrt{x^2+1}-x}{2\left[\sqrt{x^2+1}\,(1+x^2)-x(x^2+1)\right]}$$
$$=\frac{\sqrt{x^2+1}-x}{2\left[(1+x^2)\left(\sqrt{x^2+1}-x\right)\right]}=\frac{1}{2(1+x^2)}$$

Section 3.8

6. $f(x)=\log_{10}\left(\dfrac{x}{x-1}\right)=\log_{10}x-\log_{10}(x-1) \Rightarrow f'(x)=\dfrac{1}{x\ln 10}-\dfrac{1}{(x-1)\ln 10}$ or $-\dfrac{1}{x(x-1)\ln 10}$

12. $h(x)=\ln\left(x+\sqrt{x^2-1}\right) \Rightarrow h'(x)=\dfrac{1}{x+\sqrt{x^2-1}}\left(1+\dfrac{x}{\sqrt{x^2-1}}\right)=\dfrac{1}{x+\sqrt{x^2-1}}\cdot\dfrac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}=\dfrac{1}{\sqrt{x^2-1}}$

16. $y=\ln(x^4\sin^2 x)=\ln x^4+\ln(\sin x)^2=4\ln x+2\ln\sin x \Rightarrow y'=4\cdot\dfrac{1}{x}+2\cdot\dfrac{1}{\sin x}\cdot\cos x=\dfrac{4}{x}+2\cot x$

32. $y = \ln(x^3 - 7) \Rightarrow y' = \frac{1}{x^3 - 7} \cdot 3x^2 \Rightarrow y'(2) = \frac{12}{8-7} = 12$, so an equation of a tangent line at (2, 0) is $y - 0 = 12(x - 2)$ or $y = 12x - 24$.

38. $y = \sqrt[4]{\frac{x^2+1}{x^2-1}} \Rightarrow \ln y = \frac{1}{4}\ln(x^2+1) - \frac{1}{4}\ln(x^2-1) \Rightarrow \frac{1}{y} y' = \frac{1}{4} \cdot \frac{1}{x^2+1} \cdot 2x - \frac{1}{4} \cdot \frac{1}{x^2-1} \cdot 2x \Rightarrow$
$y' = \sqrt[4]{\frac{x^2+1}{x^2-1}} \cdot \frac{1}{2}\left(\frac{x}{x^2+1} - \frac{x}{x^2-1}\right) = \frac{1}{2}\sqrt[4]{\frac{x^2+1}{x^2-1}}\left(\frac{-2x}{x^4-1}\right) = \frac{x}{1-x^4}\sqrt[4]{\frac{x^2+1}{x^2-1}}$

42. $y = (\sin x)^x \Rightarrow \ln y = x \ln(\sin x) \Rightarrow \frac{y'}{y} = x \cdot \frac{1}{\sin x} \cdot \cos x + [\ln(\sin x)] \cdot 1 \Rightarrow y' = (\sin x)^x [x \cot x + \ln(\sin x)]$