Section 2.9

4. **(a)** $(a)' = $ II, since from left to right, the slopes of the tangents to graph (a) start out negative, become 0 , then positive, then 0 , then negative again. The actual function values in graph II follow the same pattern.

(b) $(b)' = $ IV, since from left to right, the slopes of the tangents to graph (b) start out at a fixed positive quantity, then suddenly become negative, then positive again. The discontinuities in graph IV indicate sudden changes in the slopes of the tangents.

(c) $(c)' = $ I, since the slopes of the tangents to graph (c) are negative for $x < 0$ and positive for $x > 0$, as are the function values of graph I.

(d) $(d)' = $ III, since from left to right, the slopes of the tangents to graph (d) are positive, then 0 , then negative, then 0 , then positive, then 0 , then negative again, and the function values in graph III follow the same pattern.

12.

24.
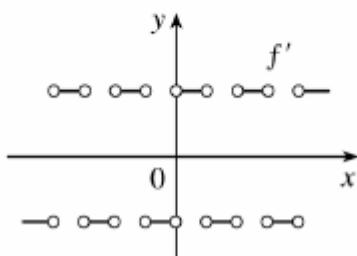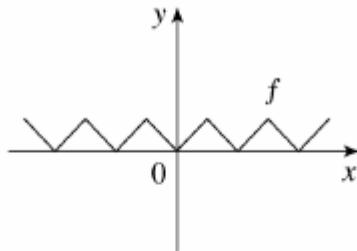
$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{[5(x+h)^2+3(x+h)-2]-(5x^2+3x-2)}{h}$$

$$= \lim_{h \to 0} \frac{5x^2+10xh+5h^2+3x+3h-2-5x^2-3x+2}{h} = \lim_{h \to 0} \frac{10xh+5h^2+3h}{h}$$

$$= \lim_{h \to 0} \frac{h(10x+5h+3)}{h} = \lim_{h \to 0}(10x+5h+3)=10x+3$$

Domain of f = domain of f' = R.

30.

$$g'(x) = \lim_{h \to 0} \frac{g(x+h)-g(x)}{h} = \lim_{h \to 0} \frac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h} = \lim_{h \to 0} \frac{x^2-(x+h)^2}{h(x+h)^2 x^2}$$

$$= \lim_{h \to 0} \frac{x^2-(x^2+2xh+h^2)}{h} = \lim_{h \to 0} \frac{-2xh-h^2}{h(x+h)^2 x^2} = \lim_{h \to 0} \frac{-2x-h}{(x+h)^2 x^2} = \frac{-2x}{x^4} = -2x^{-3}$$

Domain of g = domain of g' = $\{x \mid x \neq 0\}$.

Section 3.1

6. $F(x)=-4x^{10} \Rightarrow F'(x)=-4(10x^{10-1})=-40x^9$

10. $f(t)=\dfrac{1}{2}t^6-3t^4+t \Rightarrow f'(t)=\dfrac{1}{2}(6t^5)-3(4t^3)+1=3t^5-12t^3+1$

18. $y=\sqrt[3]{x}=x^{1/3} \Rightarrow y'=\dfrac{1}{3}x^{-2/3}=\dfrac{1}{3x^{2/3}}$

22. $y=\sqrt{x}\,(x-1)=x^{3/2}-x^{1/2} \Rightarrow y'=\dfrac{3}{2}\,x^{1/2}-\dfrac{1}{2}\,x^{-1/2}=\dfrac{1}{2}\,x^{-1/2}(3x-1)$ [factor out $\dfrac{1}{2}\,x^{-1/2}$]

or $y'=\dfrac{3x-1}{2\sqrt{x}}$.

28. $y=ae^{v}+\dfrac{b}{v}+\dfrac{c}{v^{2}}=ae^{v}+bv^{-1}+cv^{-2} \Rightarrow y'=ae^{v}-bv^{-2}-2cv^{-3}=ae^{v}-\dfrac{b}{v^{2}}-\dfrac{2c}{v^{3}}$

30. $u=\sqrt[3]{t^{2}}+2\sqrt{t^{3}}=t^{2/3}+2t^{3/2} \Rightarrow u'=\dfrac{2}{3}\,t^{-1/3}+2\left(\dfrac{3}{2}\right)t^{1/2}=\dfrac{2}{3\sqrt[3]{t}}+3\sqrt{t}$

40. $y=(1+2x)^{2}=1+4x+4x^{2} \Rightarrow y'=4+8x$. At $(1, 9)$, $y'=12$ and an equation of the tangent line is $y-9=12(x-1)$ or $y=12x-3$.

Section 3.2

4. By the Product Rule, $g(x)=\sqrt{x}\,e^{x}=x^{1/2}e^{x} \Rightarrow g'(x)=x^{1/2}(e^{x})+e^{x}\left(\dfrac{1}{2}\,x^{-1/2}\right)=\dfrac{1}{2}\,x^{-1/2}e^{x}(2x+1)$.

8. $f(t)=\dfrac{2t}{4+t^{2}} \overset{\text{QR}}{\Rightarrow} f'(t)=\dfrac{(4+t^{2})(2)-(2t)(2t)}{(4+t^{2})^{2}}=\dfrac{8+2t^{2}-4t^{2}}{(4+t^{2})^{2}}=\dfrac{8-2t^{2}}{(4+t^{2})^{2}}$

10. $Y(u)=(u^{-2}+u^{-3})(u^{5}-2u^{2}) \overset{\text{PR}}{\Rightarrow}$

$Y'(u)=(u^{-2}+u^{-3})(5u^{4}-4u)+(u^{5}-2u^{2})(-2u^{-3}-3u^{-4})$

$=(5u^{2}-4u^{-1}+5u-4u^{-2})+(-2u^{2}-3u+4u^{-1}+6u^{-2})=3u^{2}+2u+2u^{-2}$

14. $y=\dfrac{t^3+t}{t^4-2} \overset{QR}{\Rightarrow} y'=\dfrac{(t^4-2)(3t^2+1)-(t^3+t)(4t^3)}{(t^4-2)^2}=\dfrac{(3t^6+t^4-6t^2-2)-(4t^6+4t^4)}{(t^4-2)^2}$

$$=\dfrac{-t^6-3t^4-6t^2-2}{(t^4-2)^2}=-\dfrac{t^6+3t^4+6t^2+2}{(t^4-2)^2}$$

20. $y=\dfrac{\sqrt{x}-1}{\sqrt{x}+1} \Rightarrow y'=\dfrac{(\sqrt{x}+1)\left(\dfrac{1}{2\sqrt{x}}\right)-(\sqrt{x}-1)\left(\dfrac{1}{2\sqrt{x}}\right)}{(\sqrt{x}+1)^2}=\dfrac{\dfrac{1}{2}+\dfrac{1}{2\sqrt{x}}-\dfrac{1}{2}+\dfrac{1}{2\sqrt{x}}}{(\sqrt{x}+1)^2}=\dfrac{1}{\sqrt{x}(\sqrt{x}+1)^2}$