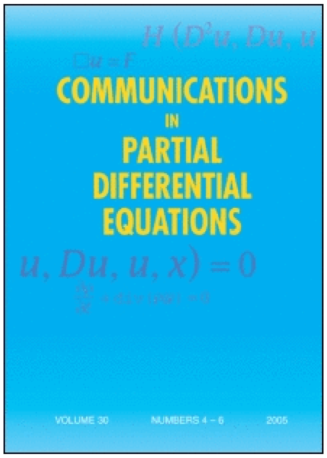

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ASYMPTOTIC STABILITY OF CRITICAL VISCOUS SHOCK WAVES FOR A DEGENERATE HYPERBOLIC VISCOUS CONSERVATION LAWS [1]

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Abstract

We study the asymptotic stability of a critical viscous shock wave for a 2×2 system of viscous conservation laws. The corresponding inviscid system is hyperbolic except at one critical state. Physical examples include the isentropic gas dynamics for van der Waals fluids. A critical shock is a shock wave with one end state being the forementioned critical state. Our main result shows that such a critical shock wave is stable under small perturbation. Further, our result is not limited to weak shock cases. A weighted energy method is adopted to prove this stability theorem. The new technical part is the introduction of a new weighted function to handle the difficulty near the critical state.

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1 Introduction

We consider the stability of viscous shock wave for the following 2×2 systems of viscous conservation laws:

$$v_t - u_x = 0, \tag{1.1}$$

$$u_t - \sigma(v)_x = \mu u_{xx}, \quad x \in R, \quad t \geq 0. \tag{1.2}$$

Here, $\mu > 0$ is the viscous coefficient. Physical systems that have this form include, for example, the viscoelasticity and the viscous isentropic gas dynamics (p-system) in Lagrangian form [1]. In the viscoelasticity, u is the velocity, v, the strain, and σ, the stress-strain function. In the viscous isentropic gas dynamics, u is the velocity, v, the specific volume, and $-\sigma(v)$ is the pressure $p(v)$. The usual assumption for σ is

$$\sigma'(v) > 0,$$

for v under consideration. In this case, the corresponding inviscid system is strictly hyperbolic. The theory for the stability of viscous shock for this case is quite complete, see [8, 12, 22] and references therein. However, for the case when $\sigma'(v) < 0$ in some region, the corresponding stability theory is incomplete. The gas dynamic equation of van der Waals fluids is one such example. The region where $\sigma'(v) < 0$ is called an elliptic region, in which all states are unstable. We call its border state (i.e. at which $\sigma'(v) = 0$) a critical state. We are insterested in the stability of viscous shocks with one end state being such a critical state. We shall call such a shock a critical shock.

For the theory of the stability of viscous shock, historically the first result was due to Il'in-Oleinik [5] for single equation with convex flux. Their proof was based on maximal principle. Many years later, Sattinger [21] gave another proof based on spectral analysis. For system cases, several energy methods in Matsumura-Nishihara [14] , Goodman [3] and Liu [10] were introduced independently to tackle this stability problem for genuinely nonlinear systems with weak shocks. It was shown that perturbations of a viscous shock cause a translation of that shock and appearance of a sequence of diffusion waves in the characteristic fields other than the shock field [10]. For a thorough result based on the weighted energy method, please see Szepessy-Xin [22]. Also, for pointwise convergence estimate, see the Green's function method by Liu [12].

For single equation with non-convex flux, Weinberger [23] obtained the stability result based on maximal principle. Jone-Gardner-Kapitula [6] gave another proof based on spectral method. Mei [17] and Matsumura-Nishihara [15] showed the stability of viscous Lax shocks and contact shocks with convergent rate estimates by a weighted energy method. See also recent work by Freistühler-Serre [2]. For system cases with non-convex fluxes (non-genuine nonlinearity), very recently Kawashima and Matsumura [8] use a weighted energy to show the stability of viscous shock for the 2×2 viscoelasticity and the 2×2 p-system with non-convex pressure function p. See also Nishihara [19], Mei-Nishihara[18] and Matsumura-Mei[13], however, their results are limited to weak shock cases, except for [13].

In this paper, we mainly adopt Kawashima and Matsumura's weighted energy method to prove the stability of a critical viscous shock. The new difficulty is the appearance of the critical state, at which the Lagrangian sound speed is zero. Previous weighted function for p-system or viscoelasticity is not applicable here. A new weighted energy function is designed to obtain a desired high order energy estimate.

Finally, due to similar technical difficulty that Kawashima and Matsumura had in the viscoelasticity [8], we also have similar restriction on the higher order derivatives of the pressure function. These conditions are carefully studied here and they cover most physical applications. Moreover, no restriction on the smallness of shock strength is required.

This paper is organized as follows. Section 2 is the reformulation of the problem and the statement of the main stability theorem. Section 3 is the proof of the stability theorem based on an *a priori* estimate. Section 4 is the proof of the *a priori* estimate. Section 5 is devoted to the application to the gas dynamics for van der Waals fluids.

Some Notation

Let L^2 and $H^l (l \geq 0)$ denote for the L^2-space and Sobolev spaces, respectively. Their norms are denoted by $\|\cdot\|$ and $\|\cdot\|_l$. Let L^2_w denote for the weighted L^2-space with the following weighted norm:

$$\|f\|_w = \Big(\int w(x)|f(x)|^2 dx\Big)^{1/2},$$

where $w(x) > 0$ is the weighted function. Similarly, $H^l_w(l \geq 0)$ denotes for the weighted Sobolev space with the weighted norm:

$$\|f\|_{l,w} = \Big(\sum_{j=0}^{l} \|\partial_x^j f\|_w^2\Big)^{1/2}.$$

We shall denote by $f(x) \sim g(x)$ if $C^{-1} g \leq f \leq Cg$ for some positive constant C. We shall also use C for a generic positive constant in our calculation.

2 Critical Viscous Shock and the Main Theorem

We consider system (1.1) and (1.2) with σ satisfying

$$\sigma'(v) > 0 \quad \text{for} \quad v > 0, \qquad \text{and } \sigma'(0) = 0 \tag{2.1}$$

and only the states $v \geq 0$ are under consideration. Suppose that at the critical point

$$\sigma'(0) = \cdots = \sigma^{(k-1)}(0) = 0, \quad \sigma^{(k)}(0) \neq 0,$$

for some integer $k \geq 2$. Then $\sigma'(v) = O(1)v^{k-1}$ for v near 0.

The characteristic speeds for the inviscid system of (1.1), (1.2) are $\lambda_{1,2} = \mp\sqrt{\sigma'(v)}$; the corresponding eigenvectors are $r_{1,2}(v, u)$. Thus, the inviscid part of the system (1.1), (1.2) is strictly hyperbolic for $v > 0$ and degenerate hyperbolic at $v = 0$.

The viscous shock that we consider here is a traveling wave solution of (1.1) and (1.2):

$$(v, u)(t, x) = (V, U)(\xi), \quad \xi = x - st,$$
$$(V, U)(\xi) \to (v_\pm, u_\pm), \qquad \xi \to \pm\infty.$$

Here, s is the shock speed, $(v_\pm, u_\pm)$ are the end states which satisfy the following Rankine-Hugoniot condition:

$$\begin{cases} -s(v_+ - v_-) - (u_+ - u_-) = 0, \\ -s(u_+ - u_-) - (\sigma(v_+) - \sigma(v_-)) = 0, \end{cases} \tag{2.2}$$

and the entropy condition [9]:

$$\frac{\sigma(v_+) - \sigma(v_-)}{v_+ - v_-} < \frac{\sigma(v) - \sigma(v_-)}{v - v_-} \tag{2.3}$$

for all v between v_- and v_+. We say a viscous shock is critical if one of its end state is the critical state 0. From the entropy condition (2.3), this critical state must be v_+ and we must have

$$v_+ = 0, \; v_- > 0, \; s > 0. \tag{2.4}$$

Remark: There are only two kinds of critical shock here: either v_+ is the critical state or v_- is. If v_- is the critical state, then the entropy condition (2.3) implies $v_+ < v_-$ and $s < 0$. In this case, we may make the following change-of-variables: $x \to -x$, $v \to -v$ and $\sigma(v) = -\sigma(v_- - v)$. Then the second case is reduced to the first case.

To find the critical viscous shock (U, V), we plug $(u, v)(x, t) = (U, V)(x - st)$ into (1.1) and (1.2), then we arrive

$$\begin{cases} -sV' - U' = 0, \\ -sU' - \sigma(V)' = \mu U''. \end{cases} \tag{2.5}$$

Integrating (2.5) and eliminating U, we obtain a single ordinary differential equation for $V(\xi)$:

$$\mu s V' = -s^2 V + \sigma(V) - a \equiv h(V), \tag{2.6}$$

where

$$a = -s^2 v_\pm + \sigma(v_\pm).$$

For the existence of viscous shock, we have the following proposition. Its proof is identical to that of Kawashima-Matsumura [8].

Proposition 1 *(Existence of viscous shocks). Suppose that (2.1) and (2.4) hold.*

(i) *If (1.1) and (1.2) admit a viscous shock wave $(V(x-st), U(x-st))$ connecting (v_-, u_-) and (v_+, u_+), then $(v_\pm, u_\pm)$ and s must satisfy the Rankine-Hugoniot condition (2.2) and the entropy condition (2.3).*

(ii) *Conversely, suppose that (2.2) and (2.3) hold, then there exists a viscous shock wave $(V, U)(x-st)$ of (1.1),(1.2) connecting (v_-, u_-) and (v_+, u_+). The viscous shock is unique up to a shift in ξ and is monotonic:*

$$u_+ > U(\xi) > u_-, \quad U_\xi(\xi) > 0, \tag{2.7}$$

$$v_+ < V(\xi) < v_-, \quad V_\xi(\xi) < 0, \tag{2.8}$$

for all $\xi \in R$. Moreover,

$$|(V, U)(\xi) - (v_\pm, u_\pm)| = O(1)e^{-c_\pm|\xi|}, \quad as\ \xi \to \pm\infty, \tag{2.9}$$

where $c_\pm = |\sigma'(v_\pm) - s|/2 > 0$ are determined constants.

Now, let us consider a perturbation of such a viscous shock at the initial time:

$$(v, u)(x, 0) = (v_0, u_0)(x), \tag{2.10}$$

where $(v_0(x), u_0(x)) \to (v_\mp, u_\mp)$ as $x \to \mp\infty$. From conservation laws, this perturbation will cause a translation and produce a diffusion wave in other characteristic field [10]. The distance of the translation x_0 and the mass m_1 of the diffusion wave can be determined by

$$\int_{-\infty}^{\infty} (v_0 - V, u_0 - U)(x)dx = x_0(v_+ - v_-, u_+ - u_-) + m_1 r_1(v_-, u_-).$$

x_0 and m_1 can be determined uniquely because $(v_+ - v_-, u_+ - u_-)$ and $r_1(v_-, u_-)$ are linearly independent. The energy estimate for the stability of viscous shock with appearance of diffusion waves can be found in Szepessy-Xin [22]. Here, we shall focus on the problem of critical state. Thus, we may assume

$$m_1 = 0 \tag{2.11}$$

for simplicity. In this case, we may also assume

$$x_0 = 0 \tag{2.12}$$

after we make a translation in x by x_0. Now, from conservation laws, we have

$$\int_{-\infty}^{\infty}(v(x,t) - V(x-st), u(x,t) - U(x-st))\,dx = (0,0)$$

for all $t \geq 0$. Thus, we may write

$$(v,u)(t,x) = (V,U)(\xi) + (\phi_\xi, \psi_\xi)(t,\xi), \quad \xi = x - st,$$

and use (ϕ, ψ) as our new unknown variables. Let us denote the initial data of (ϕ, ψ) by (ϕ_0, ψ_0). That is

$$(\phi_0, \psi_0)(x) = \int_{-\infty}^{x}(v_0 - V, u_0 - U)(y)dy.$$

To state our main theorem, we define the weighted function:

$$w(\xi) = \begin{cases} 1, & \xi \leq 0 \\ e^{b\xi}, & \xi \geq 0, \end{cases} \tag{2.13}$$

where $b = (k-1)c_+ > 0$. We also need the following two technical assumptions on σ which will be discussed in detail in the last section.

$$\sigma''(v) > 0, \text{ for } v \in [v_+, v_-], \tag{2.14}$$

$$-\frac{h(v)\sigma''(v)}{\sigma'(v)} < 4s^2, \text{ for } v \in [v_+, v_-]. \tag{2.15}$$

Condition (2.14) implies $k = 2$ in our case. Now, our main theorem can be stated as follows.

Theorem 1 *Suppose that (2.1), (2.2), (2.3), (2.11), (2.14), (2.15), $\phi_0 \in H^2$, $\phi_{0,\xi} \in H^1_w$ and $\psi_0 \in H^2_w$ hold. Then there exists a positive constant δ_1 such that if $\|\phi_0\|_2 + \|\phi_{0,\xi}\|_{1,w} + \|\psi_0\|_{2,w} < \delta_1$, then (1.1),(1.2) and (2.10) have a unique global solution $(v,u)(t,x)$ satisfying*

$$v - V \in C^0([0,\infty); H^1_w) \cap L^2([0,\infty); H^1_w),$$

$$u - U \in C^0([0,\infty); H^1_w) \cap L^2([0,\infty); H^2_w).$$

Furthermore,

$$\sup_{x\in R}|(v,u)(t,x) - (V,U)(x-st)| \to 0 \quad \text{as} \quad t \to \infty.$$

Remarks

1. Assumption (2.15) can be replaced by the following stronger condition:

$$\sigma'''(v) < 0 \text{ for } v \in [v_+, v_-]. \tag{2.16}$$

In fact, let $f(v) := s^2\sigma'(v) + h(v)\sigma''(v)$. Then $f'(v) = \sigma'(v)\sigma''(v) + h(v)\sigma'''(v) > 0$ due to $\sigma'(v) > 0$, $\sigma''(v) > 0$, $\sigma'''(v) < 0$ and $h(v) < 0$. So, $f(v)$ is monotonic increasing in $[v_+, v_-]$. Hence, $f(v) > f(v_+) = 0$, for $v \in [v_+, v_-]$. With this, we see that

$$4s^2 + \frac{h(v)\sigma''(v)}{\sigma'(v)} \geq 3s^2 > 0$$

for $v \in [v_+, v_-]$.

2. Assumption (2.15) can also be replaced by the smallness condition on the shock strength, namely, $v_- \sim v_+$. In fact, by l'Hospital rule, $\lim_{v \to v_+} \frac{-h(v)\sigma''(v)}{\sigma'(v)} = s^2$. Hence (2.15) is always true for v_- in a neighborhood of v_+.

3. In the case that v_- is the critical state, we see from the Remark of section 2 that the conditions (2.14) should be replaced by

$$\sigma''(v) < 0, \text{ for } v \in [v_+, v_-]. \tag{2.17}$$

3 Proof of the Stability Theorem

To prove the stability theorem, as in the previous works, we reformulate the problem by integrating (1.1),(1.2) in ξ and using ϕ and ψ as our new variables. The problem (1.1),(1.2) and (2.10) is then reduced to the following "integrated" system

$$\begin{cases} \phi_t - s\phi_\xi - \psi_\xi = 0 \\ \psi_t - s\psi_\xi - \sigma'(V)\phi_\xi - \mu\psi_{\xi\xi} = F\ , \\ (\phi, \psi)(0, \xi) = (\phi_0, \psi_0)(\xi) \end{cases} \tag{3.1}$$

where

$$F = \sigma(V + \phi_\xi) - \sigma(V) - \sigma'(V)\phi_\xi = O(|\phi_\xi|^2).$$

We consider the following solution space for our Cauchy problem (3.1): for any fixed $t \in (0, \infty)$ define

$$\begin{aligned} X(0,t) \;=\; & \{(\phi \in C^0(0,t;H^2), \phi_\xi \in C^0(0,t;H^1_w) \cap L^2(0,t;H^1_w), \\ & \psi \in C^0(0,t;H^2_w), \psi_\xi \in L^2(0,t;H^2_w))\} \end{aligned}$$

Then our stability theorem is a direct consequence of the following theorem.

Theorem 2 *Under the assumptions in Theorem 1, there exist positive constants δ_2 and C such that if $\|\phi_0\|_2 + \|\phi_{0,\xi}\|_{1,w} + \|\psi_0\|_{2,w} < \delta_2$, then (3.1) has a unique global solution $(\phi, \psi) \in X(0, \infty)$ satisfying*

$$\|\phi(t)\|_2^2 + \|\phi_\xi(t)\|_{1,w}^2 + \|\psi(t)\|_{2,w}^2 + \int_0^t \{\|\phi_\xi(\tau)\|_{1,w}^2 + \|\psi(\tau)\|_{2,w}^2\}\, d\tau$$
$$\leq C(\|\phi_0\|_2^2 + \|\phi_{0,\xi}\|_{1,w}^2 + \|\psi_0\|_{2,w}^2)$$

for any $t \geq 0$, and

$$\sup_{\xi \in R} |(\phi_\xi, \psi_\xi)(t,\xi)| \to 0 \quad as \quad t \to \infty.$$

Theorem 2 can be proved by a standard continuation method [7] with the use of the following local existence theorem and *a priori* estimate.

Theorem 3 *(Local Existence). Let*

$$N(t) = \sup_{0 \leq \tau \leq t} \{\|\phi(\tau)\|_2 + \|\phi_\xi(\tau)\|_{1,w} + \|\psi(\tau)\|_{2,w}\}. \tag{3.2}$$

For any $\delta_0 > 0$, there exists a positive constant t_0 depending on δ_0 such that, if $\phi_0 \in H^2$, $\phi_{0,\xi} \in H_w^1$, $\psi_0 \in H_w^2$ and $N(0) \leq \delta_0$, then the Cauchy problem (3.1) has a unique local solution $(\phi, \psi) \in X(0, t_0)$ satisfying $N(t) \leq 2\delta_0$ for $0 \leq t \leq t_0$.

Theorem 4 *(A Priori Estimates). Suppose that the assumptions in Theorem 1 hold, and $(\phi, \psi) \in X(0, t_1)$ is a solution of (3.1) for a positive t_1. Then there exist positive constants δ_3 and C which are independent of t_1 such that if $N(t) < \delta_3$, then (ϕ, ψ) satisfies*

$$\|\phi(t)\|_2^2 + \|\phi_\xi(t)\|_{1,w}^2 + \|\psi(t)\|_{2,w}^2 + \int_0^t \{\|\phi_\xi(\tau)\|_{1,w}^2 + \|\psi(\tau)\|_{2,w}^2\}\, d\tau$$
$$\leq C(\|\phi_0\|^2 + \|\phi_{0,\xi}\|_{1,w}^2 + \|\psi_0\|_{2,w}^2) \tag{3.3}$$

for $0 \leq t \leq t_1$.

The proof of the local existence theorem is standard (see, for example, [7]). The proof of the *a priori* estimate will be our main effort.

4 Proof of A Priori Estimates

There are two key lemmas to establish the *a priori* estimate (3.3). The first one is the following basic energy estimate.

Lemma 1 *It holds that*

$$\|\phi(t)\|^2 + \|\psi(t)\|_w^2 + \int_0^t \|\psi_\xi(\tau)\|_w^2\, d\tau \leq C\{\|\phi_0\|^2 + \|\psi_0\|_w^2 + N(t) \int_0^t \|\phi_\xi(\tau)\|_w^2\, d\tau\}. \tag{4.1}$$

To prove Lemma 1, we need the following lemma.

Lemma 2 *It holds that*

$$1/\sigma'(V(\xi)) \sim w(\xi) \tag{4.2}$$

$$\left|\frac{\sigma''(V)V_\xi}{\sigma'(V)}\right| \leq C, \ \textit{for all } \xi \in R, \tag{4.3}$$

Proof. From the Taylor expansion of σ near $v = 0$, we have $\sigma(v) = O(v^k)$, and $\sigma'(v) = O(v^{k-1})$. From these, (2.9) and (2.13), we see that $w(\xi)$ and $1/\sigma'(V(\xi))$ are equivalent to each other near $\xi = \infty$. On the other hand, both $w(\xi)$ and $\sigma(V(\xi))$ are bounded above zero for $\xi \in [-\infty, \bar{\xi}]$ for some sufficiently large $\bar{\xi}$. Thus, we have $0 < C_1 \leq w(\xi) \cdot \sigma'(V(\xi)) \leq C_2$ for all $\xi \in R$. For the estimation of $|\sigma''(V)V_\xi/\sigma'(V)|$, similarly, we have that it is bounded for $\xi \in [-\infty, \bar{\xi}]$. And $\sigma''(V(\xi)) = O(V(\xi)^{k-2})$ with $k > 1$ and $|s\mu V_\xi(\xi)| = |h(V(\xi))| = O(V(\xi))$ for $\xi \to \infty$. Hence, from (2.9) we also have that $|\sigma''(V)V_\xi/\sigma'(V)| = O(1)$ as $\xi \to \infty$. ∎

Proof of Lemma 1. We multiply the first equation of (3.1) by ϕ and the second one by $\psi\sigma'(V)^{-1}$ respectively, and add them to yield

$$\begin{aligned} &\{\frac{\phi^2}{2} + \frac{\psi^2}{2\sigma'(V)}\}_t - \{\frac{s\phi^2}{2} + \frac{s\psi^2}{2\sigma'(V)} + \phi\psi + \frac{\mu}{\sigma'(V)}\psi\psi_\xi\}_\xi \\ &+\frac{\mu}{\sigma'(V)}\psi_\xi^2 - \frac{s\sigma''(V)V_\xi}{\sigma'(V)^2}\psi^2 - \frac{\mu\sigma''(V)V_\xi}{\sigma'(V)^2}\psi\psi_\xi = \frac{F\psi}{\sigma'(V)}. \end{aligned} \tag{4.4}$$

By Schwarz's inequality, we note that

$$\left|\frac{\mu\sigma''(V)V_\xi}{\sigma'(V)^2}\psi\psi_\xi\right| \leq \eta\frac{\mu\psi_\xi^2}{\sigma'(V)} + \frac{1}{4\eta}\frac{\mu\sigma''(V)^2V_\xi^2\psi^2}{\sigma'(V)^3}. \tag{4.5}$$

where $0 < \eta < 1$ is a constant to be chosen later. Substituting (4.5) into (4.4) yields

$$\begin{aligned} &\{\frac{\phi^2}{2} + \frac{\psi^2}{2\sigma'(V)}\}_t - \{\frac{s\phi^2}{2} + \frac{s\psi^2}{2\sigma'(V)} + \phi\psi + \frac{\mu}{\sigma'(V)}\psi\psi_\xi\}_\xi \\ &+(1-\eta)\frac{\mu}{\sigma'(V)}\psi_\xi^2 + \frac{z(V)}{\sigma'(V)}\psi^2 \leq \frac{F\psi}{\sigma'(V)}, \end{aligned} \tag{4.6}$$

with

$$z(v) = -\frac{\sigma''(v)h(v)}{4\eta s^2\mu\sigma'(v)^2}[4\eta s^2\sigma' + h\sigma'']$$

Here, we have used $s\mu V_\xi = h(V)$. From (2.15), we can choose η such that $0 < \eta < 1$ and

$$|4(\eta - 1)s^2\sigma'(v)| \leq 4s^2\sigma'(v) + h(v)\sigma''(v)$$

for $v \in [0, v_-]$. Then we arrive

$$z(v) \geq 0, \qquad \text{for } v \in [v_+, v_-],$$

By these facts, integrating (4.6) over $[0,t]\times R$ and using that $w(\xi)\sim\sigma'(V)^{-1}$, $\sup_{(\xi,\tau)\in R\times[0,t]}|\psi(\xi,\tau)|\le CN(t)$, and

$$\int_0^t\int_{-\infty}^{\infty}\Big|\frac{F\psi}{\sigma'(V)}\Big|\,d\xi\,d\tau\le CN(t)\int_0^t\|\phi_\xi(\tau)\|_w^2\,d\tau$$

by $|F|=O(1)\phi_\xi^2$, then we arrive

$$\|\phi(t)\|^2+\|\psi(t)\|_w^2+\int_0^t\|\psi_\xi(\tau)\|_w^2\,d\tau+\int_0^t\int_{-\infty}^{\infty}z(V)\frac{\psi^2}{\sigma'(V)}\,d\xi\,d\tau$$
$$\le C\Big(\|\phi_0\|^2+\|\psi_0\|_w^2+N(t)\int_0^t\|\phi_\xi(\tau)\|_w^2\,d\tau\Big). \tag{4.7}$$

Finally, we can drop the term $\int_0^t\int_{-\infty}^{\infty}z(V)\psi^2/\sigma'(V)\,d\xi\,d\tau$ in (4.7) to obtain (4.1) because of its positivity. ∎

The following steps are to treat the energy estimates for (ϕ_ξ,ψ_ξ) in H_w^1. Differentiating (3.1) in ξ and multiplying the first equation by ϕ_ξ and the second one by $\sigma'(V)^{-1}\psi_\xi$ then adding them, we obtain

$$\Big\{\frac{\phi_\xi^2}{2}+\frac{\psi_\xi^2}{2\sigma'(V)}\Big\}_t-\Big\{\frac{s\phi_\xi^2}{2}+\frac{s\psi_\xi^2}{2\sigma'(V)}+\phi_\xi\psi_\xi+\frac{\mu}{\sigma'(V)}\psi_\xi\psi_{\xi\xi}\Big\}_\xi+\frac{\mu}{\sigma'(V)}\psi_{\xi\xi}^2$$
$$-\frac{s\sigma''(V)V_\xi}{\sigma'(V)^2}\psi_\xi^2-\frac{\mu\sigma''(V)V_\xi}{\sigma'(V)^2}\psi_\xi\psi_{\xi\xi}=\frac{F_\xi\psi_\xi}{\sigma'(V)}+\frac{\sigma''(V)V_\xi}{\sigma'(V)}\phi_\xi\psi_\xi. \tag{4.8}$$

From Lemma 2: $|\sigma''(V)V_\xi/\sigma'(V)|\le C$ and $|\sigma''(V)V_\xi|\le C$, we get by Cauchy inequality that

$$\Big|\frac{\mu\sigma''(V)V_\xi}{\sigma'(V)^2}\psi_\xi\psi_{\xi\xi}\Big|\le\frac{\mu\psi_{\xi\xi}^2}{2\sigma'(V)}+\frac{C\mu\psi_\xi^2}{2\sigma'(V)}, \tag{4.9}$$

$$\Big|\frac{\sigma''(V)V_\xi}{\sigma'(V)}\phi_\xi\psi_\xi\Big|\le\frac{\varepsilon\phi_\xi^2}{\sigma'(V)}+\frac{C\psi_\xi^2}{4\varepsilon\sigma'(V)}, \tag{4.10}$$

where $0<\varepsilon<1$ is a constant to be chosen later (by (4.27)). Substituting (4.9) and (4.10) into (4.8), integrating (4.8) over $[0,t]\times R$ and using (4.2) and (4.3) yield

$$\|\phi_\xi(t)\|^2+\|\psi_\xi(t)\|_w^2+\int_0^t\|\psi_{\xi\xi}(\tau)\|_w^2\,d\tau$$
$$\le C\Big\{\|\phi_{0,\xi}\|^2+\|\psi_{0,\xi}\|_w^2+\varepsilon\int_0^t\|\phi_\xi\|_w^2\,d\tau$$
$$+(1+\varepsilon^{-1})\int_0^t\|\psi_\xi(\tau)\|_w^2\,d\tau+\Big|\int_0^t\int_{-\infty}^{+\infty}\frac{F_\xi\psi_\xi}{\sigma'(V)}\,d\xi\,d\tau\Big|\Big\}. \tag{4.11}$$

From (4.1), we have

$$(1+\varepsilon^{-1})\int_0^t\|\psi_\xi(\tau)\|_w^2\,d\tau\le C(1+\varepsilon^{-1})\Big(\|\phi_0\|+\|\psi_0\|_w+N(t)\int_0^t\|\phi_\xi(\tau)\|_w^2\,d\tau\Big). \tag{4.12}$$

By integration by parts and (4.3), we get

$$\Big|\int_0^t\int_{-\infty}^{+\infty}\frac{F_\xi\psi_\xi}{\sigma'(V)}\,d\xi\,d\tau\Big| = \Big|\int_0^t\int_{-\infty}^{+\infty}F\Big\{\frac{\psi_\xi}{\sigma'(V)}\Big\}_\xi\,d\xi\,d\tau\Big|$$

$$\le C\int_0^t\int_{-\infty}^{+\infty}|\phi_\xi|^2\Big(\Big|\frac{\sigma''(V)V_\xi}{\sigma'(V)}\Big|\frac{|\psi_\xi|}{\sigma'(V)}+\frac{|\psi_{\xi\xi}|}{\sigma'(V)}\Big)\,d\xi\,d\tau$$

$$\le CN(t)\int_0^t(\|\phi_\xi(\tau)\|_w^2+\|\psi_{\xi\xi}(\tau)\|_w^2)\,d\tau, \tag{4.13}$$

Plugging (4.12) and (4.13) into (4.11), we have proved the following lemma.

Lemma 3 *It holds that*

$$\|\phi_\xi(t)\|^2+\|\psi_\xi(t)\|_w^2+(1-CN(t))\int_0^t\|\psi_{\xi\xi}(\tau)\|_w^2\,d\tau$$

$$\le C\Big\{(1+\epsilon^{-1})\left(\|\phi_0\|_1^2+\|\psi_0\|_{1,w}^2\right)+[\varepsilon+(1+\varepsilon^{-1})N(t)]\int_0^t\|\phi_\xi(\tau)\|_w^2\,d\tau\Big\}. \tag{4.14}$$

The next lemma is devoted to estimate $\int_0^t\|\phi_\xi(\tau)\|_w\,d\tau$. It is the second key lemma in this section.

Lemma 4 *It holds that*

$$\|\phi_\xi(t)\|_w^2+\int_0^t\|\phi_\xi(\tau)\|_w^2\,d\tau\le\frac{C}{1-CN(t)}\Big\{(1+\varepsilon^{-1})[\|\phi_0\|_1^2+\|\phi_{0,\xi}\|_w^2+\|\psi_0\|_{1,w}^2]$$

$$+[\varepsilon+(1+\varepsilon^{-1})N(t)]\int_0^t\|\phi_\xi(\tau)\|_w^2\,d\tau+\int_0^t\|\psi_{\xi\xi}(\tau)\|_w^2\,d\tau\Big\}. \tag{4.15}$$

Remark. Previous works (see [8, 13, 14, 17, 18, 19]) that obtained an energy estimate like (4.15) used $\sigma'(V)\ge C>0$ which is not valid in our case. Thus, the proof of this lemma is another key point of this paper.

Proof. From the first equation of (3.1), we have

$$\phi_{\xi t}-s\phi_{\xi\xi}-\psi_{\xi\xi}=0.$$

Multiplying this equation by $w(\xi)\phi_{\xi\xi}$ yields

$$\{w(\xi)\phi_{\xi\xi}\phi_\xi\}_t-w(\xi)\phi_{\xi\xi t}\phi_\xi-sw(\xi)\phi_{\xi\xi}^2-w(\xi)\phi_{\xi\xi}\psi_{\xi\xi}=0.$$

Then use $\phi_{\xi\xi t}=s\phi_{\xi\xi\xi}+\psi_{\xi\xi\xi}$ to obtain

$$\frac{1}{2}\{(w(\xi)\phi_\xi^2)_\xi-w'(\xi)\phi_\xi^2\}_t-\{sw(\xi)\phi_\xi\phi_{\xi\xi}+w(\xi)\phi_\xi\psi_{\xi\xi}\}_\xi$$

$$+\{\frac{s}{2}w'(\xi)\phi_\xi^2\}_\xi-\frac{s}{2}w''(\xi)\phi_\xi^2+w'(\xi)\phi_\xi\psi_{\xi\xi}=0. \tag{4.16}$$

Equation (4.16) holds for both $\xi \in (-\infty,0)$ and $\xi \in (0,\infty)$, respectively. We integrate (4.16) in ξ from $-\infty$ to 0 and use $w(\xi)=1$, $w'(\xi)=w''(\xi)=0$ for $\xi \in (-\infty,0)$ to obtain

$$\frac{d}{dt}\Big(\frac{w(\xi)}{2}\phi_\xi^2\Big)\Big|_{\xi=0} - \Big(sw(\xi)\phi_\xi\phi_{\xi\xi} + w(\xi)\phi_\xi\psi_{\xi\xi}\Big)\Big|_{\xi=0} = 0. \qquad (4.17)$$

Similarly, we integrate (4.16) in ξ from 0 to ∞ and use $w(\xi)=e^{b\xi}$, $w_1'(\xi) = bw(\xi)$ and $w_1''(\xi) = b^2 w(\xi)$ for $\xi \in [0,\infty)$, then we get

$$-\frac{d}{dt}\Big(\frac{w(\xi)}{2}\phi_\xi^2\Big)\Big|_{\xi=0} - \frac{b}{2}\frac{d}{dt}\int_0^\infty w(\xi)\phi_\xi^2\,d\xi + \Big(sw(\xi)\phi_\xi\phi_{\xi\xi} + w(\xi)\phi_\xi\psi_{\xi\xi}\Big)\Big|_{\xi=0}$$
$$-\frac{sb}{2}w(\xi)\phi_\xi^2\Big|_{\xi=0} - \frac{sb^2}{2}\int_0^\infty w(\xi)\phi_\xi^2\,d\xi + b\int_0^\infty w(\xi)\phi_\xi\psi_{\xi\xi}\,d\xi = 0. \qquad (4.18)$$

Substituting (4.17) into (4.18) and using $w(\xi)$ being continuous on R, we obtain

$$\frac{b}{2}\frac{d}{dt}\int_0^\infty w(\xi)\phi_\xi^2\,d\xi + \frac{sb}{2}w(\xi)\phi_\xi^2\Big|_{\xi=0}$$
$$+\frac{sb^2}{2}\int_0^\infty w(\xi)\phi_\xi^2\,d\xi - b\int_0^\infty w(\xi)\phi_\xi\psi_{\xi\xi}\,d\xi = 0. \qquad (4.19)$$

By the Cauchy inequality:

$$\Big|b\int_0^\infty w(\xi)\phi_\xi\psi_{\xi\xi}\,d\xi\Big| \le \frac{sb^2}{4}\int_0^\infty w(\xi)\phi_\xi^2\,d\xi + s^{-1}\int_0^\infty w(\xi)\psi_{\xi\xi}^2\,d\xi,$$

and integrating (4.19) in τ from 0 to t, dropping the positive term $\frac{sb}{2}w(\xi)\phi_\xi^2\big|_{\xi=0}$, we obtain

$$\int_0^\infty w(\xi)\phi_\xi^2(t,\xi)\,d\xi + \int_0^t\int_0^\infty w(\xi)\phi_\xi^2(t,\xi)\,d\xi$$
$$\le C\Big\{\|\phi_{0,\xi}\|_w^2 + \int_0^t \|\psi_{\xi\xi}(\tau)\|_w^2\,d\tau\Big\}.$$

From this inequality and (4.14), we get

$$\int_0^\infty w(\xi)\phi_\xi^2(t,\xi)\,d\xi + \int_0^t\int_0^\infty w(\xi)\phi_\xi^2(t,\xi)\,d\xi$$
$$\le \frac{C}{1-CN(t)}\Big\{[(1+\epsilon^{-1})[\|\phi_0\|_1^2+\|\phi_{0,\xi}\|_w^2+\|\psi_0\|_{1,w}^2]+[\varepsilon+(1+\varepsilon^{-1})N(t)]\int_0^t \|\phi_\xi(\tau)\|_w^2\,d\tau\Big\}. \qquad (4.20)$$

On the other hand, from (3.1) we have

$$\mu\phi_{\xi t} - s\mu\phi_{\xi\xi} + \sigma'(V)\phi_\xi + s\psi_\xi - \psi_t = -F. \qquad (4.21)$$

Multiply (4.21) by ϕ_ξ to obtain

$$\{\frac{\mu}{2}\phi_\xi^2\}_t - \{\frac{s\mu}{2}\phi_\xi^2\}_\xi + \sigma'(V)\phi_\xi^2 + s\phi_\xi\psi_\xi - \phi_\xi\psi_t = -F\phi_\xi. \tag{4.22}$$

From $\phi_{\xi t} = s\phi_{\xi\xi} + \psi_{\xi\xi}$, we have

$$\begin{aligned} -\phi_\xi\psi_t &= -\{\phi_\xi\psi\}_t + \phi_{\xi t}\psi \\ &= -\{\phi_\xi\psi\}_t + \{(s\phi_\xi + \psi_\xi)\psi\}_\xi - s\phi_\xi\psi_\xi - \psi_\xi^2. \end{aligned} \tag{4.23}$$

Substituting (4.23) into (4.22) yields

$$\{\frac{\mu}{2}\phi_\xi^2 - \psi\phi_\xi\}_t + \sigma'(V)\phi_\xi^2 - \{\frac{s\mu}{2}\phi_\xi^2 - s\psi\phi_\xi - \psi\psi_\xi\}_\xi = \psi_\xi^2 - F\phi_\xi. \tag{4.24}$$

Integrating (4.24) over $[0,t]\times R$, using the Cauchy inequality:

$$\begin{aligned} \Big|\int_{-\infty}^{+\infty}\psi\phi_\xi\,d\xi\Big| &\le \frac{\mu}{4}\|\phi_\xi(t)\|^2 + \mu^{-1}\|\psi(t)\|^2 \\ &\le \frac{\mu}{4}\|\phi_\xi(t)\|^2 + C\mu^{-1}\|\psi(t)\|_w^2, \end{aligned}$$

also noting $\sigma'(V(\xi)) \ge \sigma'(V(0)) > 0$ for $\xi \le 0$ (since $\sigma''(V) > 0$ and $V_\xi < 0$), we then get

$$\begin{aligned} &\frac{\mu}{4}\|\phi_\xi(t)\|^2 + \sigma'(V(0))\int_0^t\int_{-\infty}^0 \phi_\xi^2\,d\xi\,d\tau + \int_0^t\int_0^\infty \sigma'(V)\phi_\xi^2\,d\xi\,d\tau \\ &\le C\Big\{\|\phi_{0,\xi}\|^2 + \|\psi_0\|^2 + \|\psi(t)\|_w^2 + \int_0^t \|\psi_\xi(\tau)\|^2\,d\tau + N(t)\int_0^t \|\phi_\xi(\tau)\|^2\,d\tau\Big\} \\ &\le C\Big\{\|\phi_{0,\xi}\|^2 + \|\psi_0\|^2 + \|\psi(t)\|_w^2 + \int_0^t \|\psi_\xi(\tau)\|_w^2\,d\tau + N(t)\int_0^t \|\phi_\xi(\tau)\|_w^2\,d\tau\Big\}. \end{aligned} \tag{4.25}$$

Finally, (4.15) follows from (4.1), (4.20) and (4.25). ∎

Lemma 5 *It holds*

$$\begin{aligned} &\|\phi(t)\|_1^2 + \|\phi_\xi(t)\|_w^2 + \|\psi(t)\|_{1,w}^2 + \int_0^t \{\|\phi_\xi(\tau)\|_w^2 + \|\psi(\tau)\|_{1,w}^2\}\,d\tau \\ &\qquad\le C(\|\phi_0\|_1^2 + \|\phi_{0,\xi}\|_w^2 + \|\psi_0\|_{1,w}^2) \end{aligned} \tag{4.26}$$

for $N(T) \ll 1$.

Proof. Combining Lemmas 1–4, we have

$$\begin{aligned} &\|\phi(t)\|_1^2 + \|\phi_\xi(t)\|_w^2 + \|\psi(t)\|_{1,w}^2 + \int_0^t \{\|\phi_\xi(\tau)\|_w^2 + \|\psi(\tau)\|_{1,w}^2\}\,d\tau \\ &\le \frac{C}{1 - CN(t)}\Big\{(1+\epsilon^{-1})[\|\phi_0\|_1^2 + \|\phi_{0,\xi}\|_w^2 + \|\psi_0\|_{1,w}^2] + [\varepsilon + (1+\varepsilon^{-1})N(t)]\int_0^t \|\phi_\xi(\tau)\|_w^2\,d\tau\Big\}. \end{aligned}$$

Now, we choose ϵ such that

$$\frac{C\epsilon}{2} < \frac{1}{4}, \tag{4.27}$$

then choose $N(t)$ such that

$$\begin{aligned} 1 - CN(t) &\geq \frac{1}{2}, \\ \frac{C}{2}(1+\epsilon^{-1})N(t) &\leq \frac{1}{4}, \end{aligned}$$

then we obtain (4.26). ∎

The energy estimate for $(\phi_{\xi\xi}, \psi_{\xi\xi})$ can be obtained by repeating the same procedure in Lemmas 1—4. We list the result as follows and omit the details.

Lemma 6 *It holds that*

$$\begin{aligned} \|\phi_{\xi\xi}(t)\|^2 + \|\phi_{\xi\xi}(t)\|_w^2 + \|\psi_{\xi\xi}(t)\|_w^2 + \int_0^t \{\|\phi_{\xi\xi}(\tau)\|_w^2 + \|\psi_{\xi\xi\xi}(\tau)\|_w^2\}\, d\tau \\ \leq C(\|\phi_0\|_2^2 + \|\phi_{0,\xi}\|_{1,w}^2 + \|\psi_0\|_{2,w}^2) \end{aligned} \tag{4.28}$$

for $N(T) \ll 1$.

Proof of Theorem 4. Commbining Lemmas 1-6, we have proved (3.3) provided that $N(T)$ is less than a suitably small constant, say $\delta_3 > 0$. ∎

5 Application to the van der Waals Model

For applications, we consider the viscous p-system for van der Waals fluids:

$$v_t - u_x = 0, \tag{5.1}$$

$$u_t + p(v)_x = \mu u_{xx}, \quad x \in R, \quad t \geq 0, \tag{5.2}$$

where u is the velocity, v, the specific volume, p, the pressure, and $\mu > 0$, the viscous coefficient. The equation of state considered here is given by van der Waals:

$$p(v) = \frac{R\theta}{v-b} - \frac{a}{v^2}, \quad \text{for} \quad v > b, \tag{5.3}$$

where $R > 0$ is the gas constant, $\theta > 0$ the absolute temperature (assumed to be constant), and a and b are positive constants (See Figure 1).

When $R\theta b/a > (2/3)^3$, then $p'(v) < 0$ for all $v > b$ and the corresponding inviscid equation is strictly hyperbolic. The fluid is in vapor phase. In this case, the stability of viscous shock waves has been studied by Kawashima-Matsumura [8] Mei [16], and Matsumura-Mei [13] for the Lax shock case, and by Nishihara [19] and Mei-Nishihara [18] for the contact shock case (i.e. $-p'(v) = s^2$ at $v = v_-$ or v_+).

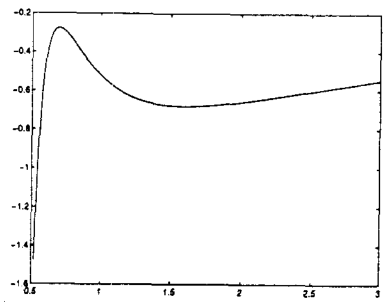

Figure 1: The graph of $\sigma(v) = -p(v)$ (5.3) with $R\theta = 2.35$, $a = 3$ and $b = 1/3$.

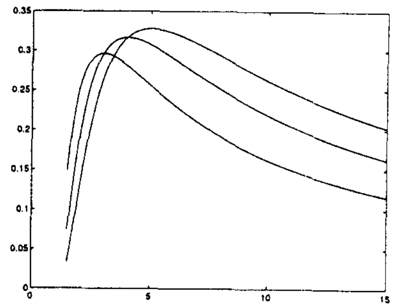

Figure 2: The graph of $f_i(m)$, $i = 1, 2, 3$

When $R\theta b/a < (2/3)^3$, there exists an interval (v_1, v_2), where $p'(v) > 0$. In this case, the corresponding inviscid equation is elliptic in this region and hyperbolic elsewhere. This equation of state is used to model fluid that exhibits water-vapor phase transition. The state in the region (v_2, ∞) is called in vapor phase, while the state in (b, v_1) the water phase. The state in the elliptic region is linearly unstable.

The van der Waals gas dynamics has been investigated by many researchers recently (see [4, 8, 11, 13, 14, 17, 18, 19] and references therein). The stability of a phase interface was obtained by Hoff [4] for the 3×3 Navier-Stokes van der Waals model. Here, our stability analysis covers the case: $R\theta b/a \leq (2/3)^3$. The critical shock we consider is in one phase (i.e. either entilely in (v_2, ∞) or in (b, v_1)), but one of its end state is the boundary state of that phase, at which the Lagrangian sound speed is zero.

We now investigate the technical conditions (2.14) and (2.15) for the van der Waals fluids.

Let us rescale v by $v = mb$. Then the derivatives of $\sigma(v)$ are given by the follows.

$$\sigma^{(i)}(v) = \Big(f_i(m) - \frac{R\theta b}{a}\Big)\frac{(-1)^i a}{(m-1)^{i+1} b^{i+2}}, \tag{5.4}$$

$$f_i(m) = \frac{(i+1)(m-1)^{i+1}}{m^{i+2}} \quad i = 1, 2, 3 \tag{5.5}$$

for $m \in (1, \infty)$. The graphs of $f_i(m)$ $(i = 1, 2, 3)$ are plotted in Figure 2.

When $R\theta b/a < (2/3)^3$, each f_i intersects $y = R\theta b/a$ at only two points: m_{ij}, $j = 1, 2$. They satisfy the following ordering relation (see Figure 2):

$$m_{11} < m_{21} < m_{31} < m_{12} < m_{22} < m_{32}.$$

Regarding to the sign of $\sigma^{(i)}$, let $v_{ij} = m_{ij}b$, $i = 1,2,3$, $j = 1,2$, from (5.4) and (5.5), we have for $i = 1,2,3$ that

$$(-1)^i\sigma^{(i)}(v)\begin{cases} > 0 & \text{for } v \in (v_{i1}, v_{i2}) \\ < 0 & \text{otherwise.}\end{cases} \tag{5.6}$$

This equation of state models water-vapor phase transition. The region (b, v_{11}) is the water-phase region, (v_{12}, ∞), the vapor-phase region, and (v_{11}, v_{12}), the water-vapor mixed region. The inviscid part of (5.1) and (5.2) becomes elliptic in (v_{11}, v_{12}). It is easy to see this region is linearly unstable by simple Fourier method.

Our first interesting region is the region (v_{12}, v_{22}), where $\sigma'(v) > 0$, $\sigma''(v) > 0$, $\sigma'''(v) < 0$ and v_{12} is the critical point. We see that our Theorem 1 is applicable in this vapor-phase region. For the water-phase region, we see that in (b, v_{11}), $\sigma'(v) > 0$ and $\sigma''(v) < 0$. Unfortunately, $\sigma'''(v) > 0$ which is in wrong sign. However, as we have mentioned in Remark 2 of Theorem 1 that we can always find a state $\bar{v}$ in a neighborhood of v_+ such that (2.15) is satisfied in region $(\bar{v}, v_{11})$.

When $R\theta b/a = (2/3)^3$, we see that $\sigma'(v) = 0$ only at the point $v = 3b$, and $\sigma'(v) > 0$ otherwise. This model has water and vapor phases but no water-vapor mixed region. In this case we can see from Figure 2 that

$$v_{11} = v_{12} = v_{21} = 3b < v_{31} < v_{22} < v_{32}.$$

We then obtain that Theorem 1 is applicable for the water region $(b, 3b)$ as well as a vapor region $(3b, \bar{v})$, where $\bar{v}$ is a neighboring state of $3b$.

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