**Honour List of Mathematical Logic**

**Karl Weierstrass** (1815-1897): made analysis rigorous by the use of logic; by using complex combinations of *quantifiers*.

**Richard Dedekind** (1831-1916): introduced the axiomatic treatment of the number systems: natural numbers, integers, rational numbers, real numbers. All other number systems are based on that of the natural numbers. The system of the natural numbers is completely described by the categorical second order "Peano" axioms.

"Was sind and was sollen die Zahlen ?" (1887) ("What are and what should be the numbers?")

**Georg Cantor** (1845-1918): founder of set theory. Cantor's diagonal argument has become the source of many results of mathematical logic.

**Gottlob Frege** (1848-1925): discovered a complete system of logic (an extension of first-order logic), and applied it to establishing a formal system of classes (sets), with the intention of formalizing the totality of mathematics. The founder of the foundational school of logicism in the philosophy of mathematics.

"Begriffschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens " (1879) ("Concept writing: a formula language of pure thought modeled on that of arithmetic").

"Die Grundlagen der Arithmetic" (1884) ("The foundations of arithmetic")

"Grundgesetze der Arithmetic" (1893 and 1903) ("The basic laws of arithmetic").

**Giuseppe Peano** (1845-1932): in "Formulaire de mathematiques" (1895, 1908), he introduced the modern language of logic.

**David Hilbert** (1862-1932): in response to the so-called crisis of the foundations (see below at Bertrand Russell), established the program of *metamathematics*, the mathematical, and preferably constructive, study of the formal aspects of axiomatic systems. Hilbert clarified the precise definition of first-order logic (1917 and 1928). By "Hilbert's thesis", we mean the idea that all axiomatic systems can be, and are ultimately to be, expressed in first-order logic.

**Ernst Zermelo** (1871-1953): discovered the axiom of choice as an indispensable axiom of set theory. Essentially established what we today call the Zermelo-Fraenkel axiom system of set theory, denoted ZFC, with "C" standing for the axiom of choice. ZFC has been, essentially universally, accepted as the official foundational system for all of (pure) mathematics.

**Bertrand Russell** (1872-1970): discovered the so-called "Russell's paradox" in Frege's logical system for classes (in the "Grundgesetze": see above), and thereby precipitated the so-called "crisis of the foundations". Discovered and described the theory of types.

"Principles of Mathematics" (1903).

"Principia Mathematica" (with A. N. Whitehead; three volumes:1910, 1912 and 1913).

**L. E. J. Brouwer** (1881-1966): critic of set theory and classical logic. The founder of the foundational school of intuitionism in the philosophy of mathematics. Intuitionistic logic appears (unexpectedly) as the internal logic of the mathematical structures called toposes (**Alexander Grothendieck, Willliam Lawvere**). Intuitionistic set theory and recursion theory connect in an unexpected validation of Church's thesis (see below) (**William Powell, Martin Hyland**).

**Alfred Tarski** (1901-1983): established formal semantics of logical systems. He made the model theoretic view the basic approach in several branches of logic.

**Alonzo Church** (1903-1995): announces Church's thesis, giving a precise definition of computability ("calculability" in Shoenfield's word).

**John von Neumann** (1903-1957): foundations of set theory (1928); pioneer of the *programmable* computer.

**Kurt Godel** (1906-1978): the "greatest logician since Aristotle": the person who showed in a final manner that the world of abstract concepts is open-ended, and cannot be grasped in its totality by the human mind. His completeness theorem and his incompleteness theorems (1930 and 1931) are the main results of mathematical logic. Godel's work clarifies in several unexpected aspects what can and what cannot be done in the framework of Hilbert's program of metamathematics.

**Stephen Cole Kleene** (1909-1994): establishes modern recursion theory.

"Introduction to Metamathematics" (1952)

**Alan Turing** (1912-1954): the abstract machine he introduced, called today the Turing machine, makes the Church-Turing thesis plausible, and became the basis of the modern computer.

**Paul Cohen** (1934-2007): proves (1962) the independence of the continuum hypothesis from ZFC (see above at Zermelo) (establishing the truth or falsehood of the continuum hypothesis is the first of Hilbert's twenty-three problems presented in the year 1900), as well as the independence of the axiom of choice from the rest of the axioms of ZFC. He obtained the Fields Medal for his work (1964).