Sample questions for exams: second installment

(See the “first installment” for general remarks.)

[15] Use Egyptian techniques (without shortcuts) to multiply 10 by 5, 100 by 50, 50 by 100, and divide 460 by 20.

[16] Quote from our text: “… since the Babylonian number system was a place value system, the actual algorithms for addition and subtraction, including carrying and borrowing, may well have been similar to modern ones.”

Note: $[a_n, a_{n-1}, ..., a_0; a_{-1}, a_{-2} ...]_{60}$ denotes

$$a_n \cdot 60^n + a_{n-1} \cdot 60^{n-1} + ... + a_0 \cdot 60^0 + a_{-1} \cdot 60^{-1} + a_{-2} \cdot 60^{-2} + ... \ .$$

If there is no semicolon “;”, then we have an integer: $a_{-1} = a_{-2} = ... = 0$.

Carry out the following addition and subtraction in base 60:

$[43,50;59,47]_{60}$ + $[35,8;0,31,27]_{60}$; $[59,0,0,0]_{60}$ − $[35,8;0,31,27]_{60}$.

[17] 1) First, perform the following multiplication in base 3 (at this point, do not convert the numbers into the decimal system!):

$[1 \ \ 2 \ \ 1 \ \ 2]_3 \times [1 \ \ 2]_3$;

and, 2) second, convert the numbers involved into decimal expressions, and check your first result.

[18] 1) Perform the Egyptian multiplication $[\overline{6} \ \ \overline{8} \ \ \overline{12}] \times [\overline{2} \ \ \overline{3} \ \ \overline{4}]$ directly (without conversions) (remember that an Egyptian fraction cannot have a repeated unit fraction $\overline{a}$), and 2) then, with any method that you can think of, show that the result can also be written in the form $[\overline{a} \ \ \overline{b} \ \ \overline{c}]$.

[19] Suppose a, b, e and f are positive integers, $d = \gcd(a,b)$, $c = \mathrm{lcm}(a,b)$. Write down: 1) the relation that exists between the numbers a, b, d, and c;

2) all the facts and logical relations that you can think of that take place concerning and between the following statements (about which we are not making any additional assumptions):

$e \mid a,\ \ e \mid b,\ \ e \mid d,\ \ d \mid a,\ \ d \mid b,\ \ a \mid c,\ \ b \mid c,\ \ a \mid f,\ \ b \mid f,\ \ c \mid f$.

(A possible *fact* about statement A is that "A is true"; a possible *relation between* statements A, B and C is that "A and B imply C".)

Give justifications, in the form of stating relevant general facts (without proofs).

[20] 1) Give a description, using algebraic notation if necessary, of the Euclidean algorithm, and why its result (you should clearly indicate how the computing agent recognizes that the computation has arrived at the result!) is the greatest common divisor of the integers one starts with.

2) Show that $\gcd(ca, cb) = c \cdot \gcd(a, b)$.

[21] Prove that, for any given integers a, b and e, there is an integer d which has the following properties: 1) $d \mid a,\ \ d \mid b,\ \ d \mid e$; and 2) whenever $f \mid a,\ \ f \mid b,\ \ f \mid e$, then also $f \mid d$.