

Sample questions for exams: first installment

(More will be added later. The present list covers only a part of the geometry we considered. The numbering does not follow any logical order.)

Note: Proofs asked for should be based on facts (axioms, theorems) stated but not proved. Those facts should be chosen in a reasonable manner. You should not try prove everything from axioms; this would require several “lemmas”, and would make the proof too long (for us now). On the other hand, you should not give a proof of a theorem using another one which is a trivial variant, or obviously a stronger form, of the one to be proved. Indeed, your answer to the question will be judged not simply on the basis whether or not the proof is *correct*, but also, on the issue whether your proof is *meaningful*.

- [1] Prove that a parallelogram is a rhombus if and only if its diagonals are perpendicular.
- [2] Prove that a parallelogram is a rhombus if and only if one of its diagonals bisects the angles of the parallelogram at its endpoints.
- [3] True or false?: ***If*** in a quadrilateral $ABCD$, with sides AB, BC, CD, DA , the opposite sides are equal in length, $AB = CD$, $BC = DA$, ***then*** the quadrilateral $ABCD$ is a parallelogram. Give sufficient reasons for your answer.
- [4] Give a proof of the Pythagorean theorem using areas (but no similarity).
- [5] Give a proof of the Pythagorean theorem using similarity (but no areas).
- [6] State Euclid’s parallel postulate (Elements, Book I, Postulate 5). You may use your own words, but do not change the contents of the original. A statement that is equivalent to, but differs essentially in content from, Euclid’s original is not an acceptable answer.
- [7] Describe the construction of an equilateral triangle. Discuss what is missing in Euclid’s statement as to the possibility of the construction.
- [8] State the three congruence criteria for triangles.
- [9] Prove that in an equilateral triangle, all (inner) angles are equal.
- [10] Let $ABCDE$ be a regular pentagon inscribed in a circle with radius r , and $\hat{A}\hat{B}\hat{C}\hat{D}\hat{E}$ another one, inscribed in a circle with radius \hat{r} . Prove that $AB : r = \hat{A}\hat{B} : \hat{r}$.
- [11] Prove that for a triangle ABC , sides AC and BC are equal: $AC = BC$ ***if and only if*** the angles $\sphericalangle A$ and $\sphericalangle B$ are equal: $\sphericalangle CAB = \sphericalangle CBA$.

[12] Prove that for any triangle, there is a unique circle (the “circumcircle”) that passes through each of the three vertices of the triangle.

[13] Prove that the sum of the inner angles of any quadrilateral is equal to 360° .

[14] Let $ABCD$ be a convex quadrangle: all inner angles are $< 180^\circ$. Prove that $ABCD$ is cyclic (there is a circle passing through all four points A, B, C, D) **if and only if** opposite angles are supplementary: $\angle ABC + \angle CDA = 180^\circ$.