MATH 338/History and Philosophy of Mathematics/Fall 2009

Assignment 1

Due: Monday, September 14

Part 1. Do Exercises **2, 6, 8, 10, 18, 20, 21, 24, 26, 28** in Chapter 1, on pages 28 and 29 in the text.

Part 2. Answer the following questions concerning Egyptian geometry:

What is the Egyptian formula
for the area of the circle of diameter d ;
for the volume of the cylinder of height h, and circular base of diameter d ;
for the volume of a truncated square-based pyramid, with appropriate length data for the base, the top and the height ;
for a hemispherical basket whose opening is a disk of diameter d ?

Instructions:

1. First, read, at least superficially, the whole text of Chapter 1. This means reading 27 pages, much of which is non-technical. Then for each problem, find the corresponding part of the text, and use it to work on the problem.

2. Please make sure that *all your calculations take place on integers only*. (This rule will be in force later as well.) This means that all numerical results should be given as integers, or ordinary fractions: in the form $\frac{p}{q}$, or possibly $-\frac{p}{q}$, with p and q positive integers.

Non-exact decimal approximations of rational numbers (0.333 for 1/3, for instance) *are unacceptable.* Decimal expressions are OK if they are exact: 0.25 is OK for ¼, but not preferable to ¼.

Calculators may be used, but only for operations on integers. For instance, when doing the addition $\frac{p}{q}+\frac{r}{s}$, one should calculate the numerator $p \cdot s + q \cdot r$ and the denominator

$q \cdot s$, form the fraction $\frac{p \cdot s + q \cdot r}{q \cdot s}$, and reduce it to lowest terms (by dividing down with the greatest common factor). Often, this can be done by finding common factors *before* calculating the integers $p \cdot s + q \cdot r$ and $q \cdot s$.

3. The above restrictions require some changes to **problem 24**: "Convert the Babylonian approximation 1;24,51,10 to $\sqrt{2}$ to decimals and determine the accuracy of the approximation" . Here, first the given base-60 fraction should be converted into an *exact* ordinary fraction $\frac{p}{q}$. Then the value $2 - \left(\frac{p}{q}\right)^2$ should be calculated, again as a simple fraction. Finally, note that $2 - a^2 = (\sqrt{2} - a) \bullet (\sqrt{2} + a)$, and use this to estimate $\left|\sqrt{2} - \frac{p}{q}\right|$ in the form $\left|\sqrt{2} - \frac{p}{q}\right| < r \cdot 10^{-s}$ with some integers $r < 10$ and s. The most important point here is that we are not using any modern calculation of $\sqrt{2}$ to give the desired result.