Math 240 – MIDTERM

Monday, Oct 22, 2012, 18h00-19h30

The exam is 90 minutes long and contains 3 questions. Write your answers clearly in the notebook provided. There are a total of 20 points possible.

Read questions carefully before answering. You may quote any result/theorem seen in the lectures without proving it (unless, of course, it is what the question asks you to prove). State any result/theorem you use from the lectures.

1. (Logic, Circuits.)
   (a) (1 point) What is the depth of the circuit in Figure 1?
   (b) (3 points) Write down the logical formula computed by the circuit in Figure 1. (For this question, 1 is “True” and 0 is “False”.)

   ![Figure 1](image)

   (c) (2 points) Check using a truth table whether the logical formula from part (b) is a tautology (i.e. whether the circuit always outputs “1”).

2. (Proofs, Induction.) (4 points) Prove by induction that \( \binom{4n+3}{2} \) is odd for all integers \( n \geq 0 \).

3. (Number Theory.)
   (a) (2 points) Find \( \text{GCD}(133, 296) \). Keep track of your steps; it will be useful in part b).
   (b) (3 points) Find integers \( s \) and \( t \) so that \( 296s + 133t = \text{GCD}(133, 296) \).
   (c) (2 points) Find a positive integer \( x < 296 \) so that \( 133x \equiv 2 \pmod{296} \).
   (d) (3 points) What is \( 133^{611} \pmod{29} \)?