Math 240 – Assignment 3


1. *(Irrational numbers).* Prove that $\sqrt{7}$ is irrational.

2. *(Euclid’s algorithm).* Use Euclid’s algorithm to find
   
   (a) $\gcd(583, 297)$
   (b) $\gcd(1208, 252)$
   (c) $\gcd(55, 34)$

3. *(Congruences).* Evaluate each of the following (showing your work).
   
   (a) $17^{2012} \pmod{13}$
   (b) $12^{1729} \pmod{36}$

4. *(Modular equations).* Solve each of the following for the variable $x$ (showing your work).
   
   (a) $5x + 16 \equiv 0 \pmod{29}$
   (b) $232x \equiv 12 \pmod{599}$

5. *(Pigeonhole principle).*
   
   (a) Given a subset $X \subseteq \{1, \ldots, 10\}$, define $s(X) = \sum_{x \in X} x$. For example, $s(\{1, 5, 8\}) = 1 + 5 + 8 = 14$. Prove that if we choose any 28 subsets of $\{1, \ldots, 10\}$, each containing at least one and at most 3 elements, there are two of these subsets $A$ and $B$ such that $s(A) = s(B)$.
   
   (b) Prove that if $n + 1$ integers are selected from $\{1, \ldots, 2n\}$ then the selection includes integers $a$ and $b$ such that $a | b$. 