

# CURRICULUM VITAE

**John P. Labute**

(Revised April 12, 2010)

## 1 GENERAL INFORMATION

Date of Birth: February 26, 1938

Citizenship: Canadian

Degrees:

1. B.Sc.(Honours) University of Windsor (1960)
2. M.A. Harvard University (1961)
3. Ph.D. Harvard University (1965). Title of thesis: *Classification of Demuskin Groups* (written under the direction of John Tate and J.-P. Serre)

## 2 Special Awards and Fellowships

1. Woodrow Wilson Fellowship (1960/61)
2. Harvard Arts and Sciences Fellowship (1961-63)
3. Research Assistantship, Harvard University (1964/65)
4. NRC Postdoctorate Overseas Fellowship: College de France (1965-67)
5. Canada Council Leave Grant: Institute for Advanced Study (1975/76)

## 3 Positions Held

1. Lecturer, University of Windsor (1963/64)
2. Assistant Professor, McGill University (1967-70)
3. Associate Professor, McGill University (1970-2007)
4. Retired Professor, McGill University (2008-)

## 4 RESEARCH

### 4.1 Research Interests

My main research interests stem from papers of Shafarevich and his student Demuškin on the galois groups of  $p$ -extensions of local and global fields. Serre, in his 1963 Seminar Bourbaki exposé of a paper of Demuškin on the galois group of the maximal  $p$ -extension of a local field, gave a cohomological description of the groups discovered by Demuškin. He also extended the results of Demuškin and introduced an new invariant in the hopes of completing the classification of these ‘Demuškin groups’. In the Fall of 1964, while Serre was on leave at Harvard, I completed the classification of Demuškin groups using the Lie algebra structure on the lower  $p$ -central series. I published these results in the Canadian Journal of Mathematics and it is the standard reference on the subject.

During the next two years, while holding an NRC Overseas Fellowship at the Collège de France, I developed the theory of Demuskin groups of countable rank and partially answered a question of Serre on the cohomological dimension of one-relator pro- $p$ -groups; this work subsequently appeared in the *Inventiones Mathematicae*. His original question as to whether a one-relator pro- $p$ -group was of cohomological dimension one if the relator was not a proper power was proved to be false by Gildenhuys when he gave an example of a relator which was not a  $p$ -th power and the group with this relator having an element of finite order. But the question as to whether a pro- $p$ -group with a single defining relator without an element of order  $p$  is of cohomological dimension 2 still remains unanswered and has motivated a large part of my research activity.

In 1968, while at McGill, I became aware of the work of Magnus and his students on the lower central series of discrete one-relator groups and immediately realized that my work in my *Inventiones* paper could be applied to answer completely the questions that they had only obtained partial results on. In a paper, which appeared in 1970 in the *Journal of Algebra*, I determined completely the Lie algebra associated to the lower central series of a one-relator pro- $p$ -group when the initial form of the relator was not a  $p$ -th power.

The next several years were spent in expanding my interests into number theory, algebraic geometry and algebraic K-theory. While I have only one joint paper with Russell on  $K^2$  of truncated polynomial rings, my students produced two Master’s theses and three Ph.D. theses. In 1972, I returned to Serre’s question by trying to understand  $p$ -th powers in free pro- $p$ -groups. In 1975, while on leave at the IAS in Princeton, I wrote two papers on this subject. The first paper determined the annihilator of homogeneous element in a free Lie algebra under the adjoint representation and was to be a key result for the paper of Berman on Kac-Moody algebras in characteristic  $p$ . This was also a key result for later results of Koch-Kukkuk-Labute on the determination of the Lie algebra of the lower central series of a Demuskin group and for generalizations to the case of several relators by Abramov. The second paper used the results of the first to determine the Lie algebra associated to the lower central series of the free product of a cyclic group of order  $p$  and a free pro- $p$ -group. While at Princeton, Iwasawa got me interested in the galois group of the maximal unramified  $p$ -extension of the extension of the rationals obtained by adjoining a  $p$ -th root of unity. These groups have the interesting property that every subgroup of finite index has a finite abelianization and Iwasawa wanted to know what I could say about them—possibly giving a classification of such groups. After several years of joint effort with Kisilevsky, we obtained some results on the question of finiteness which were subsequently published in 1987 in the *Proceedings of the Laval Number Theory Conference*.

In 1980 I began to generalize my work of the determination of the Lie algebra associated to central series of one-relator groups to the case of several relators. This work was finished while on leave at Harvard in 1982/83 and was published in the *Transactions of the AMS* in 1985. During this leave I also wrote a joint paper with D. Dummit on a new characterization of Demuškin groups in response to a question posed by K. Iwasawa. This paper appeared in the *Inventiones Mathematicae*

in 1983. The next several years were devoted to applications and extensions of my work in the case of several relators. This resulted in two papers which appeared in 1990. One of these papers gave a new proof of Murasugi's conjecture for link groups in the strengthened form of Massey-Traldi. The equivalence with the connectedness of the linking diagram modulo  $p$  for all primes  $p$  gave an important clue in our later results on the structure of Galois groups of maximal  $p$ -extensions of  $\mathbb{Q}$  with tame ramification at a finite set of primes.

Due to the work of Bass and Lubotsky on linear central filtrations I renewed my interest in one-relator Lie algebras. The determination of the center of one-relator Lie algebras was crucial to their work and I was able to determine exactly when such Lie-algebras over a field have a non-trivial center and also in the graded case over a PID. What happens in the non-homogeneous case over a PID is still an open question and one that I am actively pursuing. The culmination of this work came when I was able to prove that graded one-relator Lie algebras over a PID have a free ideal of finite corank. This is the deepest structure theorem for one-relator Lie algebras that has been proven to date. It is hoped that this result can be used to determine the structure of the Lie algebra of derivations of a one-relator Lie algebra. Little is known about this Lie algebra.

During my leave in 1993 I was also invited to visit H. Koch in Berlin to collaborate in determining the Lie algebra associated to the lower central series of a Demuskin group. Together with his student S. Kukkuk we were able to solve this question using methods of my earlier papers. Some of these results are in the Ph.D. Thesis of Kukkuk which was accepted in July 1995. These and further results of mine appeared in a joint paper with Koch and Kukkuk published in *Acta Arithmetica*. These results have been extended to the case of several relators by G. Abramov, a Ph.D. student of Helmut Koch modulo a strong independence condition on the defining relators.

In a recent paper I have discovered a new class of pro- $p$ -groups with cohomological dimension 2 with the same number of generators and relations and have shown that they appear strikingly often as Galois groups of  $p$ -extensions of  $\mathbb{Q}$  with restricted tame ramification when  $p$  is odd. Not much was known about these groups except that they were not  $p$ -adic analytic by Golod-Shafarevich when the number of ramified primes was at least 4 and that subgroups of finite index had finite abelianizations (the FAb property). That these groups could be of cohomological dimension 2 and, in particular, torsion free was totally unexpected to the people working in the field such as Helmut Koch and Nigel Boston. In 1969, Kuzmin had asked whether there were pro- $p$ -groups of cohomological dimension 2 with the same number of generators and having finite abelianization. By the Fontaine-Mazur Conjecture, the Galois group of the maximal  $p$ -extension unramified outside a finite set of primes not containing  $p$  has  $p$ -adic analytic quotients. Proving or disproving this would have important implications for the conjecture. In this direction, we have just produced the first conjectural examples of non-analytic pro- $p$ -groups having the FAb property with  $p$  odd that have an explicit presentation. This we did by truncating a Koch presentation for the Galois group. The hope is that certain of these groups are isomorphic to the corresponding Galois group much like in the case of Demuskin groups.

I am also collaborating with Jan Minac to prove a conjecture of his on the structure of the Galois group of the maximal  $p$ -extension of a field. We worked on this conjecture in May 2003 in his BIRS Research Group Project and were able to use my results on  $p$ -extensions to prove the conjecture in some important special cases. In another direction we have, together with Nicole Lemire and John Swallow, given a vast generalization of the Schreier Formula to Galois Cohomology. We also have obtained a new characterization of Demuskin groups.

I am also working on the problem of finding an algorithm to determine the strong freeness of  $m$  Lie polynomials of degree 2 in  $m$  variables. Such criteria are the own means at present for determining the cohomological dimension of the Galois group of the maximal  $p$ -extension of  $\mathbb{Q}$  unramified outside a set of finite set of primes not containing the odd prime  $p$ . Together with Michael Bush of the University of Massachusetts at Amherst, we have obtained an algorithm for strong freeness in the case  $m = 4$ . The case  $m > 4$  remains open.

## 4.2 Research publications in refereed journals

1. J. Minac, J. Labute, *Mild pro-2-groups and 2-extensions with restricted ramification* (preprint).
2. D. Karagueuzian, J. Minac, J. Labute, *The Bloch-Kato Conjecture and Galois Theory*, Ann. Sci. Math. Québec (to appear).
3. J. Labute, *Fabulous pro-p-groups*, Ann. Sci. Math. Québec 32 (2008), no 2, 189-197.
4. M. Bush and J. Labute, *Mild pro-p-groups with 4 generators*, J. Algebra 308 (2007), 828-839).
5. J. Labute, N. Lemire, J. Minac, J. Swallow, *Cohomological Dimension and Schreier's Formula in Galois Cohomology*, Canad. Math. Bull. 50 (2007), no 4, 588-593.
6. J. Labute, *Mild Groups and Galois Groups of p-Extensions of  $\mathbb{Q}$* , J. Reine Angew. Math. 596 (2006).
7. J. Labute, N. Lemire, J. Minac, J. Swallow, *Demuskin groups, Galois modules, and the elementary type conjecture*, J. Algebra 304 (2006), 1130-1146.
8. H. Koch, S. Kukkuk, J.P Labute, *Nilpotent Local Class Field Theory*, Acta Arithmetica, LXXXIII.1 (1998), pp. 45-64.
9. John P. Labute, *Free Ideals of One-Relator Graded Lie Algebras*, Transactions of the AMS, Vol 347, No 1, Jan 1995, pp. 175-188.
10. J.P. Labute, *The Lie Algebra Associated to the Lower Central Series of Link Groups and Murasugi's Conjecture*, Proceedings of the American Mathematical Society, 109 (1990), pp. 951-956.
11. J.P. Labute, *The Determination of the Lie Algebra Associated to the Lower Central Series of a Group*, Trans. AMS, 288 (1985), pp. 51-57.
12. J.P. Labute and D. Dummit, *On a new characterization of Demuskin Groups*, Invent. Math. 73 (1983), pp. 413-418.
13. J.P. Labute, *Free Lie Algebras as Modules over their Enveloping Algebras*, Proc. AMS, 68 (1978), pp. 135-139.
14. J.P. Labute, *The Lower Central Series of the Group  $\langle x, y : x^p = 1 \rangle$* , Proc. AMS, 66 (1977), pp. 197-201.
15. J.P. Labute and P. Russell, *On  $K_2$  of Truncated Polynomial Rings*, J. Pure and Applied Algebra, 6 (1975), pp. 239-251.
16. J.P. Labute, *On the Descending Central Series of Groups with a Single Defining Relation*, Journal of Algebra, 14 (1970), pp. 16-23.
17. J.P. Labute, *Algèbres de Lie et pro-p-groupes définis par une relation*, Invent. Math., 4 (1967), pp. 142-158.
18. J.P. Labute, *Classification of Demuskin Groups*, Can. J. Math., 19 (1967), pp. 142-158.
19. J.P. Labute, *Demuskin groups of type  $\aleph_0$* , Bull. Soc. Math. France, 94 (1966), pp. 211-244.
20. J.P. Labute, *Les groupes de Demuskin de rang dénombrable*, C.R. Acad. Sc. Paris, t. 262 (1966), pp. 4-7.
21. J.P. Labute, *Classification des groupes de Demuskin*, C.R. Acad. Sc. Paris, t. 260 (1965), pp. 1043-1046.

### 4.3 Research Publications in Refereed Conference Proceedings

1. J.P. Labute, *The Lie Algebra Associated to the Lower Central Series of a Free Product of Cyclic Groups of Prime Order  $p$* , Proceedings of the AMS Special Session in Combinatorial Group Theory– Infinite Groups, held April 23-24, 1988, Contemporary Mathematics, 109 (1990), pp. 91-98.
2. J.P. Labute and H. Kisilvesky, *On a Sufficient Condition for for the  $p$ -Class Tower of a CM-field to be Infinite*, Proceedings of the 1987 Laval Number Theory Conference, Number Theory, J.-M De Koninck & C. Levesque (ed) (1989), pp. 554-560, Walter de Gruyter, New York.

### 4.4 Expository Articles in Refereed Publications

1. J.P. Labute, *Groups and Lie Algebras: The Magnus Theory*, Contemporary Mathematics, Vol 169, 1994 pp. 397-406.

### 4.5 Research Monographs

1. J.P. Serre (with the collaboration of W. Kuyk and J.Labute), “Abelian  $\ell$ -adic Representations and Elliptic Curves”, Benjamin (1968).

### 4.6 Publications Resulting from Research by Ph.D. and M.Sc. Students and Postdoctoral Fellows Supervised

1. L. Simons, *The Hilbert symbol for tamely ramified Abelian extensions of 2-adic number fields*, Manuscripta Mathematica, vol. 58, pp. 345-362 (1987).
2. A. Pianzola, *On the Arithmetic of the Representation Ring and Elements of Finite Order in Lie Groups*, Journal of Algebra, vol. 108, pp. 1-33 (1985).
3. A. Pianzola, *Generators and Relations for Real Forms of Some Kac-Moody Algebras*, Communications in Algebra, vol. 15, pp. 935-959 (1987).
4. A. Pianzola, *On Automorphisms of Semisimple Lie Algebras*, Algebras, Groups and Geom., vol 2, pp. 95-116 (1985).
5. B. Saint-Donat, *Variétés de translation et théorème de Torelli*, C.R. Acad. Paris (1975).
6. J. Graham, *On Continuous  $K_2$  of Formal Power Series Fields*, Alg. K-Theory II, Classical algebraic K-theory and connections with arithmetic, Proc. Conf. Battelle Mem. Inst. Seattle (1972), pp. 474-486; Lecture Notes in Math # 342, Springer Verlag (1973).
7. D. Rideout, *A simplification of the formula for  $L(1, \chi)$  where  $\chi$  is a totally imaginary Dirichlet character of a real quadratic field*, Acta Arithmetica Vol. 23 (1973), pp. 329-327.

### 4.7 Non-refereed Articles in Journals and Conference Proceedings

1. J.P. Labute, *The Lie Algebra Associated to the Lower Central Series of a Group*, Proceedings of the 1985 St. Andrews Group Theory Conference, Cambridge Univ. Press. (1986), pp. 242-245.

#### 4.8 Conference Proceedings Edited

1. Proceedings of the 1985 Montreal Number Theory Conference, (J. Labute and H. Kisilevsky, editors), Canadian Math. Society Conference Proceedings, No. 8, AMS (1987).

#### 4.9 Invited Conference Presentations

1. *Tame pro-p-groups*, AMS Special Session on Representation Theory and Galois Cohomology in Number Theory, Davidson, NC, March 3-4 (2007).
2. *Tame Galois groups*, Quebec-Maine Number Theory Conference, Sept 30 - Oct 1, 2006.
3. *Mild pro-p-groups and p-extensions of  $\mathbb{Q}$* , Oberwolfach Workshop on Pro-p-extensions of Global Fields and Pro-p-groups, May 21-27, 2006.
4. *p-Extensions of  $\mathbb{Q}$  with restricted tame ramification*, Quebec-Maine Number Theory Conference, October 1-2, 2005.
5. *Lie Algebras and Central Series of Groups*, Workshop on General Combinatorial Group Theory, CRM (Montreal), April 5-11, 1997.
6. *Free Ideals of One-Relator Lie Algebras*, International Conference on Group Theory, Spetses (Greece), July 12-23, 1993.
7. *Lie Algebras and Central Series of Groups*, AMS Group Theory Special Session (Univ. of Maryland), April 23-24, 1988.
8. *On a Sufficient Condition for the p-class Tower of a CM Field to be Infinite*, Laval Number Theory Conference, July 1987.
9. *The Lie Algebra Associated to the Lower Central Series of a Group*, Group Theory Conference (University of St. Andrews), August 1985
10. *Lie Algebras and Central Series of Groups*, Conference on Infinite Group Theory (Iraklion, Crete), August 1984.
11. *On a New Characterization of Demuskin Groups*, Journées Arithmétiques (Unviversity of Leiden), August 1983.
12. *Lie Algebras and Central Series of Groups*, CMS Combinatorial Group Theory Special Session, June 1983.
13. *The Lie algebra Associated to the Lower Central Series of a Group with One Relator*, Combinatorial Group Theory Conference, Univ. of Waterloo, 1971.
14. *Pro-p-groups avec une relation*, Journées Arithmétiques (Grenoble), 1967.

#### 4.10 Invited Colloquia and Seminar Talks

1. *FAbulous pro-p-groups*, McGill-Laval-Vermont Number Theory Seminar, April 2007.
2. *Mild Lie algebras*, University of Western Ontario, June 2005
3. *Mild pro-p-groups and Galois groups of p-extensions of  $\mathbb{Q}$  with restricted ramification*, University of Amherst, May 2005.
4. *Mild pro-p-groups and Galois groups of p-extensions of  $\mathbb{Q}$  with restricted ramification*, IMPA Brazil, March 2005.

5. *Galois Groups of  $p$ -Extensions of  $\mathbb{Q}$  with Restricted Ramification*, McGill-Laval-Vermont Number Theory Seminar, January 2005.
6. *Computing Central Series of Link Groups and Galois Groups*, Colloquium Talk, University of Western Ontario, 2004
7. *One Relator Groups and Lie Algebras*, Max-Planck-Gesellschaft, Berlin 1995.
8. *The Lie Algebra Associated to the Lower Central Series of a Demuškin Group*, McGill-Laval-Vermont Number Theory Seminar, April 1993.
9. *Characterization of Demuškin Groups*, University of Illinois at Urbana, 1983.
10. *Les Nombres  $p$ -adiques; pourquoi?*, University of Laval, 1981.
11. *Lie Algebras and Central Series of Groups*, University of Toronto, 1981.
12. *Lie Algebras and Central Series of Groups*, University of Thessaloniki, 1976.
13. *Demuškin Groups*, Institute for Advanced Study, 1976.
14. *The Lower Central Series of the group  $\langle x, y : x^p = 1 \rangle$* , New York University, 1975.
15. *Lie Algebras and Central Series of Groups*, York University, 1972.
16. *Lie Algebras and Central Series of Groups*, Carlton University, 1972.
17. *The Lower Central Series of One Relator Groups*, Deutsche Akademie der Wissenschaften, 1969.
18. *Fonctions  $L$  des corps quadratiques*, Université de Bordeaux, 1969.
19. *Pro- $p$ -groups of Cohomological Dimension 2*, Queen's University, 1968.
20. *Class Number Formulae*, Dalhousie University, 1968.
21. *Classification of Demuškin groups*, Queen's University, 1967.
22. *Classification des groupes de Demuškin*, Collège de France, 1966.

#### 4.11 Contributed Talks

1. *Central Series of Galois Groups over  $\mathbb{Q}$  with Restricted Ramification*, CMS Number Theory Session (June 2003 Meeting in Quebec City).
2. *The Lie Algebra Associated to the Lower Central Series of Link Groups*, CMS Lie Algebra Session (Winter Meeting in Vancouver), 1987.
3. *The Determination of the Lie Algebra Associated to the Lower Central Series of a Group*, AMS Combinatorial Group Theory Session (New York) 1983.
4. *Classification of Demuškin Groups*, Instructional Conference on Algebraic Number Theory (University of Sussex), August 1965.

#### 4.12 Memberships on organizing/scientific committees of conferences

1. Organizer of the Colloque des Sciences Mathematiques du Quebec held at McGill April 12, 1997.
2. With the aid of an NSERC Conference Grant and monies from McGill and Concordia, H. Kisilevsky and I organized the 1985 CMS Summer Conference in Number Theory. This was a two week conference with twenty invited speakers from the US and Canada. The principal speakers were Dick Gross of Harvard and Harold Stark of MIT and UCSD.
3. Program chairman for the special number theory session (in honour of E. Rosenthal) at the CMS Summer Meeting at McGill. The main speakers were Daniel Quillen and Barry Mazur.

#### 4.13 External Examiner of Theses Outside McGill University

1. Ganesh Bhandari, *Galois module structure of Milnor K-theory in characteristic p*, Ph.D. Thesis, University of Western Ontario (2005).
2. S. Pauli, *Factoring Polynomials over Local Fields*, Ph.D. Thesis, Concordia University (2001).
3. G. Abramov, *Nilpotent Class Field Theory*, Ph.D. Thesis, Humboldt University at Berlin (1998).
4. S. Kukkuk, *Graduierte nilpotente Klassenkörpertheorie*, Ph.D. Thesis, Humboldt University at Berlin (1995).
5. L. Soicher, *The Computation of Galois Groups*, Master's Thesis, Concordia University (1981).
6. H. Lee, *The Brauer Group of an Integral Scheme*, Ph.D. Thesis, Queen's University (1978).
7. B.R.D Subbarao, *On the Semisimple Rank of Artin-Schrier Curves*, Ph.D. Thesis, Queen's University (1973).

#### 4.14 Research Grants

1. NSERC Discovery Grant (2006-2011), \$9,000/yr.
2. FCAR Team Grant with D. Dummit, G. Frei, H. Kisilevsky, C. Levesque, R. Murty, P. Russell (1993/96), \$75,000/yr.
3. FCAR Center Grant, CICMA, with H. Kisilevsky, C. Lam, J. McKay, R. Ford, P. Russell, R. Murty, P. Russell, C. Levesque (1993-96) \$70,00/yr.
4. FCAR Team Grant with D. Dummit, G. Frei, H. Kisilevsky, C. Levesque, R. Murty, P. Russell (1990/93), \$53,000/yr.
5. FCAR Center Grant, CICMA, with H. Kisilevsky, C. Lam, J. McKay, R. Ford, P. Russell, R. Murty, P. Russell, C. Levesque (1990-93) \$100,00/yr.
6. NSERC Research Grant (1989/90), \$6,000
7. FCAR Team Grant with D. Dummit, G. Frei, H. Kisilevsky, C. Levesque, R. Murty, P. Russell (1987-90), \$16,500/yr.
8. NSERC Research Grant (1986-89), \$5,600/yr.
9. 8. NSERC Conference Grant with H. Kisilevsky (1985), \$19,000.

10. FCAR Team grant with G. Frei, H. Kisilevsky, C. Levesque, K. Murty, R. Murty (1984-87), \$11,900/yr.
11. NSERC Research Grant (1983-86), \$4,600/yr.
12. FCAR Team Grant with G. Frei, H. Kisilevsky, C. Levesque, R. Murty (1983/84), \$8,000.
13. NSERC Research Grant (1980-83), \$4,600/yr.
14. NSERC Research Grant (1968-80), \$1,800/yr.

#### 4.15 Undergraduate research supervision

1. C. Vincent, NSERC Summer Award (2004).
2. C. Simons, NSERC Summer Award (1990).
3. H. Darmon, NSERC Summer Award (1988).
4. S. Laroche, NSERC Summer Award (1984)
5. S. Wagon, NSERC Summer Award (1976)

#### 4.16 Graduate Student Supervision

1. Audrey Baker, *An Algorithm for the Strong Freeness of Quadratic Lie Polynomials*, M.Sc. (2006).
2. S. Alajaji, *Central Filtrations of Lie Algebras*, M.Sc. (1995).
3. T. Stefanicki, *On Subalgebras of Free Lie Algebras and on Lie Algebras Associated to the Lower Central Series of a Group*, M.Sc. 1987.
4. L. Simons, *The Structure of the Hilbert Symbol for Unramified Extensions of a 2-adic Number Field*, Ph.D 1986.
5. L. Simons, *p-Adic Analysis and p-adic intergration*, M.Sc. 1979.
6. E. Brettler, *On Absolute Galois Groups of Real Function Fields in One Variable*, Ph.D. Thesis 1973.
7. P. Houde, *Equidistribution and L-functions in Number Theory*, M.Sc. 1973.
8. J. Graham, *Continuous  $K_2$  of Formal Power Series Fields*, Ph.D. 1972.
9. D. Rideout, *On a Generalization of Stickelberger's Theorem*, Ph.D. 1970.
10. W. Dicks, *Prime ideals of a Lie algebra's universal algebra*, M.Sc. 1970.

#### 4.17 Postdoctoral Supervision

1. B. Saint-Donat, Algebraic varieties, 1972-74.
2. A. Pianzola, Lie algebras, 1983-85.

#### 4.18 Refereeing of Papers Submitted for Publication

I have refereed papers for the Canadian Journal of Mathematics, the Journal of Algebra, The Proceedings of the AMS, le Journal de l'Institut Fourier, the Bulletin of the CMS and the Journal of Pure and Applied Algebra.

#### 4.19 Reviewing of Published Papers

I have reviewed numerous published papers and books for Zentralblatt and the AMS.

#### 4.20 Additional Comments on my Work

My main research interests lie in the connection between (A) group theory and algebraic number theory via class field theory and (B) group theory and the theory of Lie algebras via central series of groups. My papers on the classification of Demuškin groups exploit these connections to complete the work of Serre, Shafarevich and Demuskin on the determination of the Galois group of the maximal  $p$ -extension of a local field and determine the structure of the  $p$ -Sylow subgroup of the Galois group of the algebraic closure of a local field. This is some of my best work.

Not far behind is my seminal work pro- $p$ -groups defined by a single relator which appeared in the *Inventiones Mathematicae* in 1967. This paper was an attempt to answer a question of Serre on the cohomology of such groups. The cohomology of these groups is still not known. The methods used in this paper have had important applications to the determination of the Lie algebra associated to the lower central series of groups with one or more relators. Previously this Lie algebra was in general known only for free groups (W. Magnus, E. Witt). These results have had important applications to the classification of link groups (R. Hain) and braid groups (T. Kohno, T. Oda, D. Anick) and to the study of the automorphism group of pro- $p$ -fundamental groups (M. Asada, M. Kanko). More recently, the importance of these results was demonstrated in a recent paper of Bass and Lubotsky on linear central filtrations.

Our 1978 paper on free Lie algebras as modules over their enveloping algebras was a key ingredient in a paper of S. Berman on the classification of Kac-Moody algebras in characteristic  $p$ . Recently, this paper and my 1977 paper on the determination of the Lie algebra of the lower central series of the group  $\langle x, y : x^p = 1 \rangle$  have furnished the key ingredients in the determination of the lower central series of a Demuškin group. This has been done jointly with H. Koch and his student S. Kukkuk. G. Abramov, a student of H. Koch, has generalized this work to the case of several relators in his 1995 Ph.D. thesis.

My 1995 paper on one-relator Lie algebras is one of my best papers. This paper was written to answer a question of Bass on the center of one-relator Lie algebras. In the process of answering this question I prove an important result on the structure of such algebras in the graded case. Over a field, they are all extensions of free Lie algebras by a finite dimensional nilpotent Lie algebra. The structure of such algebras for non-homogeneous relators is still not known although we have partial results. We hope that this result will shed some light on the structure of the Lie Algebra of Derivations of a one-relator Lie algebra.

My 2005 paper on mild groups brings together many of the methods I have developed over the last 35 years. It is one of my top three papers. Mazur's analogy between link groups and Galois groups of number fields has played a large role in obtaining our results by bringing into play the combinatorics of the linking diagram. These results represent the first new results on the structure of the Galois group  $G_S(p)$  of the maximal  $p$ -extension of a number field unramified outside a finite set of primes Galois not dividing  $p$  groups since 1963 when Golod-Shafarevich proved them infinite if the number of ramified primes was at least 4. Using Abramov's result we have produced a class of pro- $p$ -groups having explicit presentations which have all the algebraic properties of the Galois groups  $G_S(p)$  including the FAb property. The hope is that these groups include the groups  $G_S(p)$ . This would be the crowning achievement of my research program begun some 40 years ago with Iwasawa's

question on a possible classification of FAb pro- $p$ -groups with the same number of generators and relations.

## 5 TEACHING

### 5.1 Undergraduate Teaching

Almost all of my undergraduate teaching has been in algebra and number theory which is my area of specialization although I has an extensive background in analysis which has allowed me to give analysis courses at the honours level. I was asked to teach the first year honours algebra course (Algebra I) during the first two years of the restructured honours program in the 70's. During the first year I followed Godement's book "Algebra", but the second year I used my own notes which I distributed to the class as they were written. In these notes I gave axioms for abstract set theory and showed how to construct the complex numbers from the empty set. I had an extremely bright class that was very appreciative of the treatment I gave. I still have copies of these notes. One of the students was Adrian Nachman who went on to get a Ph.D. in analysis at Princeton. These first two years I, together with Roger Ringlehof, conducted a joint Problems Seminar for the students of Algebra I and Analysis I. We developed a series of problems that made use of algebra and analysis; for example, problems in combinatorial topology (fixed point problems) and problems leading to the theory of Lie Groups.

I have also given Algebra III and IV a number of times. I have used Jacobson's "Algebra I" and Dummit-Foote as texts and put a lot of stress on groups, elementary number theory and Galois theory. Some exceptional students were Henri Darmon, Peter Green (both of which went on to Harvard), Marco Gualtieri and Patrick Hayden (both of which are Rhodes Scholars).

Most of the undergraduate courses I have given involve linear algebra although I have taught calculus a dozen times, complex variables twice and math for biologists once. As a result of the large number of times I have given linear algebra, I have developed a version which emphasizes linear operators at an early stage. This approach was largely motivated by 189-223 which is given to chemists, physicist's and engineers. On looking into the needs of these students, I found that the operator approach the best one to use because it enabled me to motivate the solution of several, seemingly different problems, by one general method. Since most of the topics are treated in most linear algebra texts I usually take as a text a book that is flexible enough to permit me to vary the order of the topics. This course is very conceptual and probably one of the most difficult courses that an undergraduate has to take at the majors level.

I have given the number theory courses MATH 346/377 several times. In these course I have made use of computer software to illustrate number theoretic concepts and problems. An integral part of the course is a project of the students choosing in an area not dealt with in the course. There is a written report and an oral report made in class. Claude Crépeau was one of my students and his project was on the twin primes conjecture.

In all the courses that I give I try to challenge the students by insisting on a rigorous and careful treatment of the concepts. I try to convince students of the importance to understand what one is doing and not to accept too readily everything that the instructor tells them. I find that students resist this approach, being much more appreciative of teachers who give them recipes for doing things rather than general techniques that they can adapt to special situations. While I prepare my lectures extensively beforehand, I never use notes during the lecture. I prefer to lecture spontaneously following a general plan that I develop step by step in front of the class. Students see a real live mathematician at work. This is a risky approach as problems will occur and mistakes will be made; but the process by which these problems are solved and mistakes discovered and corrected are, in my opinion, the best possible training that a student can receive— a true apprenticeship rather than sitting through a polished and sanitized lecture read from notes or slides.

I make extensive use of the web to make my notes, solutions to assignments and other materials available to students. I have also begun to experiment with web-based course software such as WeBWorK in order to provide a greater and more timely interaction with students. I have become the resource person in the department for this software and have promoted its use by my colleagues. I have also developed WeBWorK problem sets for MATH 325 and MATH 222 and was instrumental in getting Axel Hundemer and Vojkan Jaksic to a problem sets for MATH 133. At present, mainly with the help of Axel Hundemer, we have developed WeBWorK problem for 12 courses having a total enrolment of over 2500 students per term.

While I was Associate Chair I created and supervised a HelpDesk manned by some our best students. That we could liberate these students from marking was due to the fact that fewer markers were required due to the use of WebWork. Students with problems with their courses could simply walk in and get the help they required.

In 2003 the Tomlinson University Science Teaching Project awarded a three year award in the amount of \$96,000 for the project "Web-based and Collaborative Learning Project for Mathematics and Statistics". I am the project leader and the other team members are Nilima Nigam, David Bryant, Vojkan Jaksic, Axel Hundemer, Russell Steele.

In the Fall of 2003 I participated in the University Tablet PC trial. In this trial I delivered all the lectures for my MATH 133 course on a Tablet PC. The lectures were then put on the web.

## 5.2 Graduate Teaching

The first year I came to McGill I gave a graduate level course in Algebraic Number Theory. Since this was the first time I gave such a course I followed the text "Number Theory" by Borevich and Shafarevich. Because the class was exceptional and very interested in the course, I decided to have a seminar immediately following the course in which I would invite some prominent number theorists to give lectures on their specialties. The seminar was to last two months (May, June) in which the first month were preparatory for the last month when the invited specialists would lecture. These specialists were Prof. A. Brumer and K. Iwasawa. The seminar attracted a number of other people, most notably G. Pall and H. Kisilevsky. The seminar was an enormous success and eventually led to the thesis topic of D. Rideout.

This initial seminar was continued in subsequent years with different themes each year. In subsequent years the themes were Class field theory, Zeta and L-functions, K-theory, Etale Cohomology, Algebraic surfaces, Modular forms. Some of the lectureres were J. Tate, S. Lang, J. Coates, S. Kleiman, D. Quillen, H. Bass, J-L. Verdier, P. Griffiths, P. Cohn, J. Hsia and H. Kisilevsky. Some of the lectures were also given by Bernard Saint Donat while he was my Postdoctoral student. Peter Russell was an active participant in this seminar and, when Hershy Kisilevsky came to Concordia in the mid 70's, the seminar split into a Number Theory Seminar run by Kisilevsky and myself while Peter organized the Algebraic Geometry Seminar. The fact that there was an active Number Theory Seminar was, I believe, a large factor in the decision of Ram Murty to come to McGill.

In the early 70's I gave a graduate course on algebraic curves and was invited to give this course at the University of Sherbrooke. In the late 70's I participated in the McGill/Laval exchange, giving a graduate level course on elliptic curves at the university of Laval. I also given the first year graduate course in algebra on numerous occasions as well as graduate level courses on functional analysis, algebraic topology, Lie groups and Lie algebras.

## 6 OTHER ACTIVITIES

1. 1983-85 Elected to the Board of Directors of the CMS.
2. 1970- Member of the American Mathematical Society, the Mathematical Association of America, the Canadian Mathematical Society.

3. 1990-95 Director of Students for CICMA.
4. May, 1995 Departmental Representative for the Canadian Forum on Mathematics Education sponsored by the CMS.
5. 2002-2004,2005-2007 Member FQRNT Evaluation Committee for Master's Bursaries

## **7 ADMINISTRATION**

### **7.1 Departmental Administration**

1. 1976-79 Graduate Affairs Committee, Colloquium Committee, Admissions Committee.
2. 1978-81 Nominating Committee.
3. 1983-85 Graduate Affairs Committee, Admissions Committee (Chairman).
4. 1984-87 Colloquium Committee.
5. 1988-91 Undergraduate Affairs Committee, Computer and Equipment Committee (Chairman 89/90).
6. 1990-91 Network Administrator (Responsible for networking of PC's and Unix machines) I introduced the first Sun workstation (gauss) and promoted its use in the department. I also recommended the installation of the ethernet network in our department and acted as administrator for the network after it was installed.
7. 1993-1997 Computer and Equipment Committe
8. 1997-2004 Computer and Equipment Committe (Chair)
9. 2001-2004 Associate Chair of the Department

### **7.2 University Administration**

1. 1975- Served as Pro-Dean.
2. 1994-2001 Departmental Representative for the Faculty of Science Workgroup on Instructional Computing.
3. 2002-2004 Member, SC-IST Standing Committee on Technology in Learning & Teaching.