ADDENDUM TO MILD PRO-p-GROUPS AND GALOIS GROUPS OF p-EXTENSIONS OF $\mathbb{Q}$

1. Dmitri Piontkovski has pointed out that Anick's Criterion can be used to prove the strong freeness of the sequence

$$\rho_1 = [\xi_1, \xi_3], \quad \rho_2 = [\xi_2, \xi_1] + [\xi_2, \xi_4], \quad \rho_3 = [\xi_3, \xi_4] + [\xi_3, \xi_5], \quad \rho_4 = [\xi_4, \xi_5],$$

which is given after Definition 3.20. Indeed, for the ordering $\xi_1 > \xi_2 > \xi_4 > \xi_3 > \xi_5$, the leading monomials are

$$\xi_1\xi_3,\ \xi_1\xi_2,\ \xi_4\xi_3,\ \xi_4\xi_5.$$

2. In line -9 on page 18 replace $1 - \alpha_d$ by $1 - \alpha_s$.
3. Line -9 on page 18 should be

$$\dim_{\mathbb{F}_p}(gr_n(G)) = \sum_{k=1}^{n} \frac{1}{k} \sum_{r|k} \mu(k/r)(\alpha_1^r + \alpha_2^r + \cdots + \alpha_s^r).$$

4. The relator ρ_4 at the top of page 26 should be $[\xi_4, \xi_1] + [\xi_4, \xi_2]$. The corrected sequence $\rho_1, \rho_2, \rho_3, \rho_4$ can be shown to be strongly free using results of [1]. The following line beginning with "We are unable" should be replaced by "Dmitri Piontkovski has pointed out that the following sequence is strongly free."

   Indeed, after the change of variables $\xi_4 \mapsto \xi_4 + \xi_3$, $\xi_i \mapsto \xi_i$ $(i \neq 4)$ we obtain

$$\begin{aligned}
\rho_1 &= [\xi_1, \xi_2] + [\xi_1, \xi_3],\\
\rho_2 &= [\xi_2, \xi_4],\\
\rho_3 &= [\xi_3, \xi_4] + [\xi_3, \xi_1],\\
\rho_4 &= [\xi_1, [\xi_2, \xi_4]] + [\xi_1, [\xi_2, \xi_3]].
\end{aligned}$$

   Adding ρ_1 to ρ_3 and $[\xi_1, \rho_2]$ to ρ_4, we get

$$\begin{aligned}
\rho_1 &= [\xi_1, \xi_2] + [\xi_1, \xi_3],\\
\rho_2 &= [\xi_2, \xi_4],\\
\rho_3 &= [\xi_3, \xi_4] + [\xi_1, \xi_2],\\
\rho_4 &= [\xi_1, [\xi_2, \xi_3]]
\end{aligned}$$

   whose leading monomials for the ordering $\xi_3 > \xi_2 > \xi_1 > \xi_4$ are

$$\xi_3\xi_1,\ \xi_2\xi_4,\ \xi_3\xi_4,\ \xi_3\xi_2\xi_1.$$

   The result then follows by Anick's Criterion.

Date: September 28, 2006.

5. Unfortunately, it has been discovered that the next example on page 26 is not strongly free. In fact, the relators add up to zero and this appears to be true for all such sets S.

References

[1] M. Bush and J. Labute, *Mild Pro-p-Groups with* 4 *generators* (to appear in J. Algebra).