

McGill University
Math 325B: Differential Equations
Midterm Test

1. Find the general solution of the differential equation

$$x \frac{dy}{dx} = y + 2xy^2.$$

Find the solution y with $y(1) = 1$. What is the largest interval on which this solution can be defined?

2. Show that the differential equation $M + Ny' = 0$ has an integrating factor μ which is a function of xy if

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{xM - yN}$$

is a function of xy and use this to find the general solution of the differential equation

$$xe^x \frac{dy}{dx} = xye^x - ye^x + y^2.$$

3. Find the orthogonal trajectories of the family of curves $y = Cx^4$.
4. Solve the initial value problem

$$y' = x^2 + xy, \quad y(0) = 1$$

using Picard iteration. If y_n is the n -th Picard iteration, give an estimate for $|y(x) - y_n(x)|$ on the interval $[-h, h]$.