1. (a) Find all solutions of the differential equation

$$2xy' - y^2 + 1 = 0$$

   and sketch the solution curves.

   (b) For what pairs (a, b) does the initial value problem

$$2xy' - y^2 + 1 = 0, \quad y(a) = b,$$

   have (a) a unique solution, (b) more than one solution, (c) no solution?

2. Solve the initial value problem

$$(x^3 + 1)y' + x^2y - 2xy^2 - 1 = 0, \quad y(0) = -1,$$

   using the fact that there is an integrating factor which is a function of $x - y$.

3. Find all solutions of the differential equation

$$y''' - y'' + 4y' - 4y = \sin(x) + e^x.$$

4. Solve the initial value problem

$$\frac{dx}{dt} = -4x - 2y - 1,$$
$$\frac{dy}{dt} = x - y - 2,$$

   with $x(0) = 1, y(0) = -1$. What is the equilibrium solution?

5. Using Laplace transforms, solve the initial value problem

$$y'' + 3y' + 2y = \begin{cases} 0, & 0 \leq t < 1, \\ 1, & 1 \leq t < \pi, \\ \sin(t), & \pi \leq t, \end{cases}$$

   with $y(0) = 1, y'(0) = -1$.

6. (a) Show that $x = 0$ is a regular singular point of the differential equation

$$x^2y'' + (x^2 - x)y' + y = 0, \quad (x > 0).$$

   Find the indicial equation for this regular singular point.

   (b) Find the general solution of the above differential equation.