McGill University
Math 325B: Differential Equations

Assignment 8: due Thursday, April 4, 2002

1. (a) Compute the Laplace transforms of the functions

$$te^{2t}\cos(5t), \quad \sin 3t \cos 5t.$$

   (b) Find the inverse Laplace tranforms of the functions

$$\frac{s^2 + s + 1}{s^3 - s^2 - s + 1}, \quad \frac{s+1}{(s^2+1)^3}.$$

2. Using Laplace transforms, solve the initial value problem

$$y'' + y = \sin(t), \quad y(0) = y'(0) = 1.$$

3. Using Laplace transforms, solve the system

$$\frac{dx}{dt} = -2x + 3y + 1,$$
$$\frac{dy}{dt} = 2x - y - 1$$

   with the initial conditions $x(0) = 1, y(0) = -1$. Is this system stable?

4. Using Laplace transforms, solve the initial value problem

$$y'' + y' - 2y = f(t), \quad y(0) = 1, y'(0) = 2,$$

   where

$$f(t) = \begin{cases} 1, & 0 \le t < 1, \\ 0, & 1 \le t < \pi, \\ \cos(t), & \pi \le t. \end{cases}$$