McGill University
Math 325B: Differential Equations

Assignment 5: due Tuesday, February 19, 2002

1. Show that the initial value problem

$$y' = x^2 - y^2, \quad y(0) = 0$$

has a unique solution on the interval $[-1, 1]$. Find the third Picard iteration y_3 and find an estimate for $|y - y_3|$ on $[-1, 1]$.

2. Find the n-th Picard iteration y_n for the initial value problem

$$y' = 1 + x^2 y, \quad y(0) = 0.$$

Show that y_n converges to a solution y of this problem and find a formula for y. Find $y(1)$ to 2 decimal places.

3. Show that the initial value problem

$$y' = 1 + \frac{1}{1 + y^2}, \quad y(0) = 0$$

has a unique solution y which is defined for all x. Compute the second Picard iterations y_2. How good an estimate is y_2 to y on $[-h, h]$? What value of $h > 0$ will make $|y(x) - y_2(x)| \leq 1/100$ for $|x| \leq h$.