**189-265A: Advanced Calculus**

Tutorial Problem Set 5: Surface Integrals

1. Using a suitable parametrization, compute the areas and centroids of the surfaces
   (a) the graph of $z = \sqrt{1-(x^2+y^2)},\ x^2+y^2 \le 1$;
   (b) the part of the cone $z = 2\sqrt{x^2+y^2}$ where $1 \le z \le 3$;
   (c) the cylinder $x^2+y^2=4, 0 \le z \le 3$;
   (d) the part of the plane $3x+2y+z=1$ that lies inside the cylinder $x^2+y^2=9$;
   (e) the part of the plane $3x+2y+z=1$ that lies inside the cylinder $x^2+4y^2=9$;
   (f) the part of the sphere $x^2+y^2+z^2=1$ that lies above the cone $z=\sqrt{3(x^2+y^2)}$;
   (g) the part of the hemisphere $x^2+y^2+z^2=36, z \ge 0$, that lies inside the cylinder $x^2+y^2=6y$;
   (h) the square with corners at $(1,1,2),(-1,1,2),(-1,-1,2)$ and $(1,-1,2)$;
   (i) the rectangle with corners at $(1,3,4),(-2,3,4),(-2,3,-1)$ and $(1,3,-1)$;
   (j) the triangle with vertices at $(0,0,0),(-1,2,3)$ and $(2,-1,5)$;
   (k) the triangle with vertices at $(1,1,1),(0,3,4)$ and $(3,0,6)$;
   (l) the part of the cylinder $x^2+y^2=9$ for which $0 \le z \le x+3$,
   (m) the spiral ramp or helicoid obtained by joining the point $(\cos t, \sin t, 2t)$ — on a helix — to $(0,0,2t)$ where $0 \le t \le 2\pi$, [*not a nice integral!*]
   (n) the part of the hyperboloid $x^2+y^2-z^2=1$ for which $-2 \le z \le 2$,
   (o) the part of the sphere $x^2+y^2+z^2=4$ for which $-\sqrt{2} \le z \le \sqrt{2}$.

2. Calculate $\iint_S \varphi(x,y,z)dS$ if
   (a) $\varphi(x,y,z) = x^2+y$ and $S$ is the cone $z=2\sqrt{x^2+y^2},\ x^2+y^2 \le 4$;
   (b) $\varphi(x,y,z) = z$ and $S$ is the surface $z=4(x^2+y^2), x^2+y^2 \le 1$;
   (c) $\varphi(x,y,z) = x^2$ and $S$ is the spiral ramp in 1(m).

3. Calculate the flux of the given vector fields $\overrightarrow{F}$ over the given surfaces $S$.
   (a) $S$ is the unit sphere $x^2+y^2+z^2=1$ with the normal outward and $\overrightarrow{F}(x,y,z)=\vec{i}$;
   (b) $S$ is the surface made up of the hemisphere $x^2+y^2+z^2=1, z \le 0$ and the disc $x^2+y^2 \le 1, z=0$ — the boundary of half of a ball of radius 1 — (again with the normal outward) and $\overrightarrow{F}(x,y,z) = x\vec{i}+y\vec{j}+z\vec{k}$;
   (c) $S$ is the boundary of the cube $C=[0,1]\times[0,1]\times[0,1]$ — with corners at $(0,0,0)$, $(1,0,0)$, $(1,1,0)$, $(0,1,0)$ and $(0,0,1)$, $(1,0,1)$ ,$(1,1,1)$, $(0,1,1)$ — with inward normal and $\overrightarrow{F}(x,y,z)=(x,y,0)$;
   (d) $S$ is the cone $z=2\sqrt{x^2+y^2},\ x^2+y^2 \le 4$ with the standard parametrization (which means that the normal points into the cone) and $\overrightarrow{F}=\vec{k}$;
   (e) $S$ is the cone $z=2\sqrt{x^2+y^2},\ x^2+y^2 \le 4$ with the standard parametrization and $\overrightarrow{F}=(x^2,y^2,z^2)$;
   (f) $S$ is the sphere $x^2+y^2+z^2=1$ and $\vec{F}(x,y,z)=(xz,yz,0)$;
   (g) $S$ is the cylinder $x^2+z^2=4, 0 \le y \le 1$ and $\overrightarrow{F}(x,y,z)=(x,y,z)$;
   (h) $S$ is the surface of the sphere $x^2+y^2+z^2=R^2$ with outward normal and $\overrightarrow{F}=\nabla(-1/r)$, where $r^2=x^2+y^2+z^2$. [Note that the answer does not depend upon $R$. This will be important when discussing the divergence theorem.]