**189-265A: Advanced Calculus**

**Assignment 5 (due Tuesday December 4, 2001)**

1. Compute the flux of the curl of $\overrightarrow{F} = (y^3, -x^3, z^3)$ across the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$, by reducing it to the integral over the disk that makes the hemisphere a closed surface.

2. Consider a cylindrical can, open at both ends, having an unspecified curve $C_1$ as the top component of its boundary and unit circle with center the origin as bottom component. Compute

$$\oint_{C_1} \overrightarrow{F} \cdot d\overrightarrow{s},$$

where $\overrightarrow{F} = (2y^2, x^2, 3z^2)$.

3. Using the divergence theorem, compute $\iint \overrightarrow{F} \cdot d\overrightarrow{S}$, where $S$ is the surface of the cube bounded by the planes

$$x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1,$$

oriented with the outward-pointing normal and $\overrightarrow{F} = (2x - z, x^2 y, -xz^2)$. Verify your answer by computing the surface integral directly.

4. Compute the flux of the gradient of the scalar field

$$\phi(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}} + \frac{1}{\sqrt{(x-1)^2 + y^2 + z^2}}$$

inward across the surface of the parallelepiped

$$-\frac{1}{2} \leq x \leq \frac{2}{3}, \quad -1 \leq y \leq 5, \quad -\frac{1}{4} \leq z \leq 1.$$

Justify all your steps.