189-265A: Advanced Calculus

Assignment 3 (due Thursday November 1, 2001)

1. Compute $\iint_D (x+y)dxdy$, where D is the triangle with vertices located at $(2,3)$, $(1,0)$ and $(3,-1)$.

2. Let $f(x,y) = (u,v)$, where
$$u = \sin(x+y), \quad v = y^2 + x$$
and let $g(u,v) = (w,z)$, where
$$w = e^{u+v}, \quad z = u^2 + \log v.$$
If $h(x,y) = g(f(x,y)) = (w,z)$, use the chain rule in matrix form to compute
$$\frac{\partial w}{\partial x}, \quad \frac{\partial w}{\partial y}, \quad \frac{\partial z}{\partial x}, \quad \frac{\partial z}{\partial y}$$
at $(x,y) = (1,2)$.

3. Apply the implicit function theorem to show that the equations
$$xu + yvu^2 = 2,$$
$$xu^3 + y^2v^4 = 2$$
can be solved for u, v as functions of x, y near the point $(x,y,u,v) = (1,1,1,1)$. Compute the partial derivatives
$$\frac{\partial u}{\partial x}, \quad \frac{\partial u}{\partial y}, \quad \frac{\partial v}{\partial x}, \quad \frac{\partial v}{\partial y}$$
at $(1,1)$ by implicit differentiation.

4. Use the method of Lagrange multipliers to find the perimeter of the largest rectangle that can be inscribed in the ellipse $x^2 + 4y^2 = 4$.

5. Let $\overrightarrow{F} = P(x,y)\vec{i} + Q(x,y)\vec{j}$ and let
$$\overrightarrow{G}(r,\theta) = P(r\cos\theta, r\sin\theta)\vec{i} + Q(r\cos\theta, r\sin\theta)\vec{j} = \tilde{P}(r,\theta)\vec{i} + \tilde{Q}(r,\theta)\vec{j}$$
be the expression of $\overrightarrow{F}$ in polar coordinates. Using the chain rule, show that the divergence of $\overrightarrow{F}$ in polar coordinates is given by
$$\operatorname{div}(\overrightarrow{F}) = \cos\theta\frac{\partial\tilde{P}}{\partial r} + \frac{\sin\theta}{r}\frac{\partial\tilde{P}}{\partial\theta} + \sin\theta\frac{\partial\tilde{Q}}{\partial r} - \frac{\cos\theta}{r}\frac{\partial\tilde{Q}}{\partial\theta}.$$