189-265A: Advanced Calculus

Assignment 1 (due Thursday September 27, 2001)

1. Compute the integral
$$\int_C xyzds,$$
where C is the polygonal path with successive vertices at $(0,0,1)$, $(0,1,1)$ and $(1,2,3)$.

2. Compute the integral
$$\int_C (y^2+z^2)ds,$$
where C denotes the intersection of the sphere $x^2+y^2+z^2=1$ with the plane $x+y-z=0$.

3. Recall that the length of a curve C is defined as the integral
$$\int_C ds.$$
Sketch the plane curve whose equation in polar coordinates is given by
$$r = 1+\cos\theta, \quad 0 \le \theta \le 2\pi,$$
and compute its length.

4. Compute the line integral
$$\int_C 2xyzdx + x^2zdy + x^2ydz,$$
where C is the line segment from $(1,1,1)$ to $(1,2,4)$.

5. Compute the line integral
$$\int_C \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy,$$
where C is the portion of the unit circle centered at $(0,0)$, from $(1,0)$ to $(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$.

6. Sketch the planar vector field $\overrightarrow{F} = (-y, x)$. Show that the vector field $\overrightarrow{F}$ can't be a be a gradient field by finding distinct paths from some point p to a distinct point q such that the integral of $\overrightarrow{F}$ over these paths takes different values.

7. Compute the flux of the vector field
$$\overrightarrow{F} = \left(\frac{x-1}{(x-1)^2+y^2}, \frac{y}{(x-1)^2+y^2}\right),$$
across the portion of the circle of radius 1 centered at $(1,0)$ from $(2,0)$ to $(0,0)$, taken counterclockwise.