1. (a) Find parametric equations for the line passing through the point $A(0, 1, 0)$ with the direction $[1, 1, 1]$.

   (b) Find the distance of the point $Q = (1, 0, 2)$ to the line in 1(a).

2. (a) Find the normal equation of the plane $\mathcal{P}$ passing through the points

$$A(3,2,1),\ B(8,1,2),\ C(-4,1,-1).$$

(b) Find the point D where the line $\mathcal{L}$ with parametric equations

$$x = 1 + 3t, y = -1 + 2t, z = t$$

meets the plane $\mathcal{P}$ in 2(a) and find the cosine of the angle θ $(0 \leq \theta \leq \pi/2)$ between $\mathcal{L}$ and the line through D perpendicular to $\mathcal{P}$.

3. Solve the following system of linear equations by Gauss-Jordan elimination:

$$\begin{aligned} x_1 + 2x_2 + x_3 - 4x_4 &= 1 \\ x_1 + 3x_2 + 7x_3 + 2x_4 &= 2 \\ x_1 - 11x_3 - 16x_4 &= -1. \end{aligned}$$

4. (a) Let $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ be three linearly independent vectors in $\mathbb{R}^4$. If $\mathbf{u}_4$ is another vector in $\mathbb{R}^4$ which does not lie in Span($\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$), show that $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$, $\mathbf{u}_4$ are linearly independent. Identify the subspace Span($\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$, $\mathbf{u}_4$) of $\mathbb{R}^4$.

(b) A rhombus is a parallelogram all of whose sides are equal. Using vectors, show that a parallelogram is a rhombus if and only if the diagonals of the parallelogram are orthogonal.

5. (a) Find all values of a and b for which the system

$$\begin{aligned} x + 2y - bz &= 1 \\ x + 3y - z &= a \\ 2x + 5y + z &= 1 \end{aligned}$$

will have (i) a unique solution, (ii) no solution, (iii) more than one solution.

(b) Solve the system in case (iii) above.

6. Let
$$A = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}.$$

(a) Write A^{-1} as a product of two elementary matrices.

(b) Write A as a product of two elementary matrices.

7. Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

(a) Bring A to row reduced echelon form.

(b) Find bases for (i) the row space, (ii) the column space and (iii) the null space of A.

8. (a) Let A, B be 2×2 matrices with $\det(A) = 2$, $\det(B) = 3$. Find

(i) $\det(-A^3 B^{-2})$

(ii) $\det(2A^{-1}BA)$

(iii) $\det(A^{-1}A^T)$

(b) If $\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} = 1$ find

$$\begin{vmatrix} a+d & d+g & g+a \\ b+e & e+h & h+b \\ c+f & f+i & i+c \end{vmatrix}.$$

State the properties of determinants that you use in your calculation.

9. Let L be the line in R^2 with equation $2x+3y=0$ and let S, T be respectively be the transformations: reflection in L and projection onto L.

   (a) Find the standard matrices of S, T, $S \circ T$ and $T \circ S$.

   (b) Find the eigenvalues of S and T geometrically or otherwise.

10. If $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$,

(a) find the characteristic polynomial of A and the eigenvalues of A;

(b) find a basis of each eigenspace and an orthogonal matrix P such that $P^{-1}AP$ is a diagonal matrix.

11. Let $W = \text{span}\left(\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right)$

(a) Use the Gram-Schmidt process to find an orthonormal basis for W.

(b) Find a basis for $W^{\perp}$.

NAME:

STUDENT NUMBER:

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATH 133

Vectors Matrices and Geometry

Examiner: Professor V. Jaksic — Date: Monday, December 11, 2006

Associate Examiner: Professor J. Labute — Time: 14:00-17:00

INSTRUCTIONS

Attempt all questions.
All questions are of equal value.
Answer all questions on the pages provided.
Show and justify all your work.
Calculators, books and notes are not permitted.
All matrices are real matrices.

This exam comprises the cover and 16 pages with 11 questions and 5 additional blank pages.

1	2	3	4	5	6	7	8	9	10	11