Roy's maximum root and maximum canonical correlation SPMs from multivariate multiple regression analysis of imaging data⋆

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This abstract fills a gap in the random field theory for the P-value of local maxima of SPMs from multivariate linear models for image data. Examples of multivariate image data are: vector deformations to warp an MRI image to an atlas standard, diffusion tensors, and the HRF sampled at 1s intervals. Examples of multiple contrasts are: several polynomial effects, several performance measures, or differences between several groups. So far results are only available for either one variate or one contrast:

Random Field Theory results available for:		Number of contrasts, p 1	>1
Number of variates, q	1	T [1]	F [1]
	>1	Hotelling's T^2 [2]	???

For multivariate data and multiple contrasts, there are several different test statistics, all based on the eigen values r_j, $j = 1, \ldots, q$, of $W^{-1}B$, where W and B are the error and contrast mean sum of squares matrices. The natural choice is the likelihood ratio test Wilks's Λ, equivalent to the product of $1/(1 + r_j)$, but the random field theory for this has so far proved intractable [3]. However we have succeeded for an alternative, Roy's maximum root $R = \max_j r_j$, reported here.

Roy's maximum root and maximum canonical correlation

A simpler definition of Roys maximum root is the following:

1. Take a linear combination of the multivariate image data, creating univariate image data.
2. Work out the F statistic for relating the univariate image data to the multiple contrasts.
3. Roy's maximum root R is the maximum F over all such linear combinations.

The maximum canonical correlation C can be defined analogously as the maximum univariate correlation between all linear combinations of the multivariate data and multiple contrasts. The two are related by

$$R = (C^2/p)\big/((1 - C^2)/m),$$

where p and m are the degrees of freedom (df) of B and W. Applications are effective connectivity [4], detected by the maximum canonical correlation between multivariate image data at a single reference voxel, and that at all other voxels. If the reference voxel is varied as well, the auto- and cross-correlation SPMs [5] can be extended in the same way.

Random field theory

The P-value of local maxima of a smooth SPM in D dimensions is

$$\mathrm{P}\left(\max_S SPM > t\right) \approx \sum_{d=0}^{D} Resels_d(S)\ EC_d(t),$$

where $Resels_d(S)$ is the resels of the search region S and and $EC_d(t)$ is the Euler characteristic density of the SPM in d dimensions [1]. Since $\max_S R = \max_S \max_{\text{lin. comb.}} F$, we can add q extra dimensions for the linear combinations to the D dimensions of S to get a surprisingly simple form for the EC density $EC_d^R(t)$ of the Roy's maximum root SPM:

$$EC_d^R(t) = \sum_{i=0}^{q-1} Resels_i(\bigcirc_q)\ EC_{d+i}^F(t),$$

where $EC_d^F(t)$ is the EC density of the F statistic SPM with p and m df [1], and $\bigcirc_q$ is part of a unit sphere in q dimensions with

$$Resels_i(\bigcirc_q) = \left(\frac{\pi}{\log 2}\right)^{i/2} \frac{\Gamma\left(\frac{q+1}{2}\right)}{i!\left(\frac{q-1-i}{2}\right)!}$$

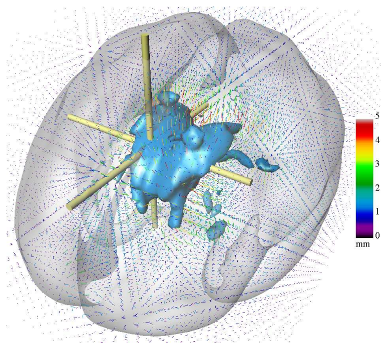
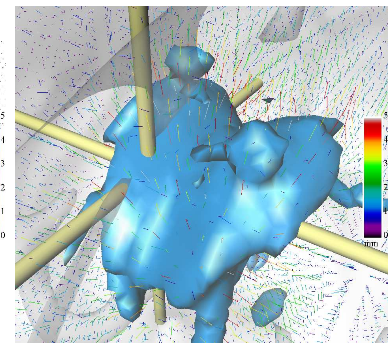
if $q - 1 - i$ is even, and 0 otherwise, $i = 0, \ldots, q-1$ [6,7].

Example: Deformation Based Morphometry

- $n_1 = 17$ patients with non-missile brain trauma who were in a coma for 3-14 days.
- $n_2 = 19$ age and sex matched controls

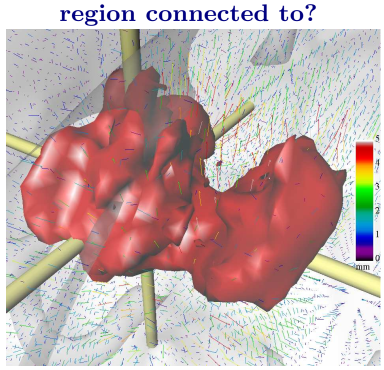
MRI images were taken after the trauma, and the multivariate data were the $q = 3$ component vector deformations needed to warp the $n = 36$ MRI images to an atlas standard, sampled on a 2mm voxel lattice. Damage is expected in white mater areas, so the search region S was defined as the voxels where smoothed average control subject white matter density exceeded 5%. For calculating the resels, this was approximated by a sphere with the same volume, 1.31cc. The effective FWHM, averaged over the search region, was 13.3mm.

Where is the damage?

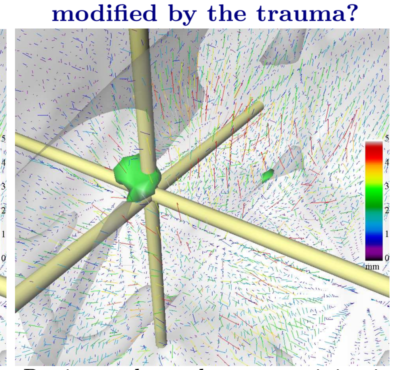
Trauma minus control average deformations (arrows and color bar), sampled every 6mm, with Hotelling's T^2 statistic for significant differences ($p = 1$, $m = 34$, $t = 54.0$, $P = 0.05$, corrected). The closeup shows that the damage is an outward movement of the anatomy, either due to swelling of the ventricles or atrophy of the surrounding white matter. The reference voxel of maximum Hotelling's T^2, used for the connectivity, is marked by the intersection of the three axes.

What is the most damaged region connected to?



Regions of effective anatomical connectivity with the reference voxel, assessed by the maximum canonical correlation C ($p = 3$, $m = 31$, $t = 0.746$, $P = 0.05$, corrected). Reference voxel is 'connected' with its neighbours (due to smoothness) and with contralateral regions (due to symmetry).

Where is the connectivity modified by the trauma?



Regions where the connectivity is different between trauma and control groups (as in [4]), assessed by Roy's maximum root R ($p = 3$, $m = 28$, $t = 30.3$, $P = 0.05$, corrected). The small region in the contralateral hemisphere is more correlated with the reference voxel in the trauma group than the control group.

References

⋆This poster is at http://www.math.mcgill.ca/keith/fmristat/poster_roy.pdf

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[7]. MATLAB code is in the FMRISTAT package at http://www.math.mcgill.ca/keith/fmristat/stat_threshold.m

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