An improved theoretical P-value for SPMs based on discrete local maxima⋆

Keith J. Worsley

Department of Mathematics and Statistics, and Montreal Neurological Institute, McGill University, Montreal, Canada

We present a new continuity correction to the P-value for local maxima of a statistical parametric map that bridges the gap between small $FWHM$, when the Bonferroni correction is accurate, and large $FWHM$, when random field theory is accurate. The new method, based on discrete local maxima, is always an upper bound (like the Bonferroni), but lower and hence more accurate for large $FWHM$, without increasing false positives. It resulted in P-values that were ∼43% lower than the best of Bonferroni or random field theory methods when applied to a typical fMRI data set.

Methods

The last step in the statistical analysis of a statistical parametric map Z is to assign P-values P to local maxima of height t. The choice is between a Bonferroni correction (BON):

$$P \le P_{\mathrm{BON}} = N \times P(Z > t) \tag{1}$$

where N is the number of voxels in the search region, or random field theory (RFT), which for large search regions has the form:

$$P \approx P_{\mathrm{RFT}} = R \times \mathrm{EC}(t) \tag{2}$$

where R is the number of resels and EC is the Euler characteristic density of the SPM [1]. The resels of a search region of volume V in D dimensions is $R = V/FWHM^D$. The proposed approximation is the expected number of discrete local maxima (DLM) above threshold:

$$P \le P_{DLM} = \sum_x P(Z > t \text{ and } Z > \text{neighbouring } Z'\text{s}), \tag{3}$$

where summation is over all voxels x in the search region. The $2D$ neighbours are those that differ by just one voxel in each lattice direction.

The DLM P-value for a Gaussian SPM only involves the correlation ρ_d of (whitened) residuals between adjacent voxels along each lattice axis d [2]. To allow for non-isotropy, average these over all voxels x in the search region to give $\bar\rho_d$ chosen so that

$$\sqrt{1-\bar\rho_d} = \sum_x \sqrt{1-\rho_d}/N, \tag{4}$$

$$\phi(z) = \exp(-z^2/2)/\sqrt{2\pi}, \quad \Phi(z) = \int_z^\infty \phi(u)du, \tag{5}$$

$$\alpha = \sin^{-1}\left(\sqrt{(1-\rho^2)/2}\right), \qquad h = \sqrt{\frac{1-\rho}{1+\rho}}, \tag{6}$$

$$Q(\rho, z) = 1 - 2\Phi(hz^+) + \frac{1}{\pi}\int_0^\alpha \exp(-\tfrac{1}{2}h^2z^2/\sin^2\theta)d\theta, \tag{7}$$

$$P_{\mathrm{DLM}} = N\int_t^\infty \left(\prod_{d=1}^D Q(\bar\rho_d, z)\right)\phi(z)dz. \tag{8}$$

• DLM is valid for almost any spatial correlation structure [2].

• Like BON, DLM is conservative, which is reassuring for practical applications, but unlike BON it is very accurate for all $FWHM$; for large $FWHM$ and thresholds DLM converges to RFT.

• A boundary correction is easily implemented. For a voxel on the boundary of the search region with just one neighbour in axis direction d, replace $Q(\rho, z)$ by $1 - \Phi(hz)$, and by 1 if it has no neighbours.

• For a non-Gaussian SPMs, such as a T-statistic SPM, Gaussianize the statistic, then adjust the $FWHM$ so that P_{RFT} of the statistic matches P_{RFT} of a Gaussian statistic, then apply the DLM method:

– Gaussianize: $Z_{\max} = \Phi^{-1}(P(T > T_{\max}))$.

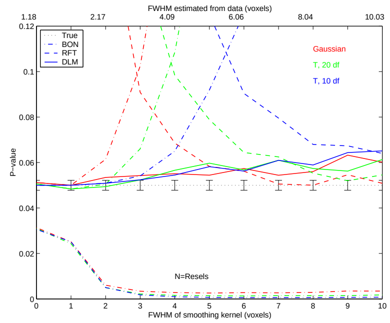
– Find $c = \mathrm{EC_T}(T_{\max})/\mathrm{EC_Z}(Z_{\max})$.

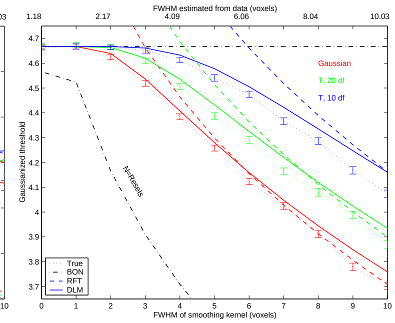
– Let $f = c^{2/D}$, and replace $\bar\rho_d$ by $|\bar\rho_d|^f$ to adjust the $FWHM$.

– Now calculate P_{DLM} as in (8) for $t = Z_{\max}$.

Results

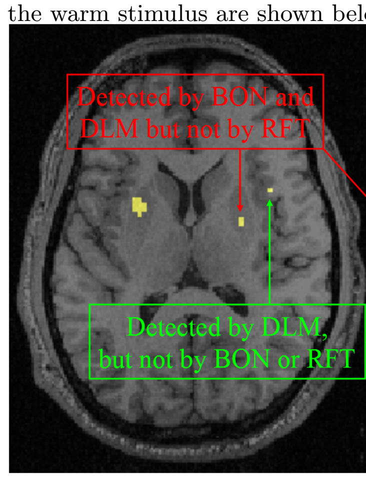
Simulated isotropic Gaussian and T-statistic SPMs with 10 and 20 degrees of freedom, $N = 32^3$ voxels and increasing $FWHM$ are shown in the Figures below (error bars show one standard deviation).





DLM is always an accurate upper (conservative) bound on the true P-value, which almost equals BON when $FWHM$=0 and slightly overestimates RFT when $FWHM$>6 voxels. In between DLM is better than either of them. The greatest discrepancy occurs at $FWHM$=3 voxels, where DLM is about half either of the others.

We compared our methods on fMRI data from one run of one subject from a study in pain perception [3]. A subject received 9s hot stimulus to the right calf, 9s rest, 9s warm stimulus, 9s rest, repeated 10 times for a total of 360s (TR=3s). 6mm smoothing was applied in-slice during motion correction to give $FWHM \sim 2.55$ voxels which puts us in the zone where BON and RFT are not very accurate. P-values for local maxima of a T-statistic (110 df) for the effect of the hot minus the warm stimulus are shown below.



$T_{\max}$	P_{BON}	P_{RFT}	P_{DLM}
5.71	0.0015**	0.0058**	0.0009**
5.29	0.0098**	0.0307*	0.0056**
5.18	0.0158*	0.0466*	0.0091**
5.17	0.0162*	0.0476*	0.0093**
5.17	0.0164*	0.0481*	0.0094**
5.15	0.0178*	0.0516	0.0102*
5.13	0.0192*	0.0549	0.0109*
5.11	0.0211*	0.0598	0.0121*
5.09	0.0227*	0.0636	0.0129*
4.92	0.0470*	0.1190	0.0267*
4.77	0.0862	0.1996	0.0487*
4.59	0.1805	0.3727	0.1014
4.54	0.2241	0.4468	0.1256

*$P < 0.05$, **$P < 0.01$

We first notice that BON is better than RFT, but DLM is better than BON, giving P-values ∼43% lower. The net result is that DLM detects one extra local maximum at $P < 0.05$, and three extra at $P < 0.01$.

Discussion

DLM is an upper bound (like Bonferroni), so if it is lower than other methods, it has to be better. It depends solely on the correlation of whitened residuals between adjacent voxels. It does not depend on isotropy, and there is a simple boundary correction. It is reasonably accurate, though still conservative, over the middle range of $FWHM$ to voxel size, the values often encountered in practice. The Bonferroni (BON) method is still more accurate for very small $FWHM$, and the random field theory (RFT) method is more accurate for large $FWHM$. To cover the whole range of $FWHM$, we recommend simply taking the best of the three methods: BON, RFT and DLM. This method has been implemented in the `FMRISTAT` and `BRAINSTAT` packages [4].

References

[1]. Worsley, K.J. *et al.* (1996). *Human Brain Mapping*, **4**:58-73.
[2]. Taylor, J.E. & Worsley, K.J. (2005). Submitted.
[3]. Worsley, K.J. *et al.* (2002). *NeuroImage*, **15**:1-15.
[4]. Taylor, J.E. (2005). *NeuroImage*, Poster #763 T-AM.

⋆#539 T-AM: poster at http://www.math.mcgill.ca/keith/fmristat/poster_dlm.pdf June 14, 2005