Detecting Changes In Non-stationary Images via Statistical Flattening

K.J. Worsley[1,2], M. Andermann[1], T. Koulis[1], D. MacDonald[2], A.C. Evans[2]
[1] *Department of Mathematics and Statistics, and* [2] *Montreal Neurological Institute, Montréal, Québec, Canada. Web: http://www.math.mcgill.ca/∼keith*

Introduction. The theoretical results for P-values of local maxima and size of supra-threshold clusters of a statistical parametric map (SPM) are not valid if the noise component of the image data is non-stationary[1]. One of the conditions for stationarity or 'flatness' (in the statistical sense) is that the FWHM should be constant across all voxels in the image. This assumption is reasonable for PET data or smoothed fMRI data, but not for two new types of image data. The first is PET or fMRI data projected onto an unfolded, inflated or flattened 2D cortical surface[2], where the different amounts of stretching of the surface alter the original constant FWHM, making it non-stationary. The second is anatomical data such as 3D binary masks of a structure[3], 2D surface displacements[4], and 3D vector deformations required to warp the structure to an atlas standard[5]. In all these cases, the smoothness of the images varies considerably from region to region, so they too are not stationary. The purpose of this abstract is to present a simple method for overcoming these problems so that random field theory can be applied to most non-stationary images.

Methods. The first step is to transform the data to a triangular (2D) or tetrahedral (3D) lattice. Data on a square or cubical lattice can be easily transformed by subdividing the squares into two triangles or the cubes into five tetrahedra. Note that no new vertices (voxels) are created, only their connectivity is altered. The second step is to estimate the *effective* FWHM (eFWHM), defined as the FWHM of a Gaussian kernel that would produce the same local smoothness of the noise component of the observed images. This is based on the normalized residuals from fitting a linear model at each voxel. If the image is stationary or 'flat' then the eFWHM should be constant; departures from this indicate non-stationarity.

A first attempt at correction is to warp the image so that the eFWHM is approximately constant. This is achieved by a local multidimensional scaling that attempts to match the new edge length to the old edge length divided by the eFWHM, so that regions with low eFWHM are stretched, and regions with high eFWHM are shrunk. The usual random field theory for P-values of local maxima and cluster sizes can then be applied to the resulting stationary or 'flattened' images. However this step is not entirely necessary. Inspection of the resulting calculations shows that the main term of the corrected P-value can be derived directly from the eFWHM image without carrying out the flattening. Essentially, Euclidean volume is replaced by integrated *resel density*, defined as $1/(\text{eFWHM})^3$, in the P-value and cluster size calculations.

Nevertheless, the entire flattened image is still necessary to find the remaining boundary correction terms for the unified P-value of local maxima, which is accurate for search regions of almost any shape or size[1]. Once again the flattening can be avoided by the following trick. We note that the P-value formula does not depend on the dimension of the space used to embed the warped image; a higher dimensional space could be used to achieve a more successful flattening. Taking this to the limit, it can be shown that *exact* flatness can be achieved by warping the data into a space whose dimensionality equals the number of images: the coordinates are just the normalized residuals. Although this cannot be visualized, the resulting boundary corrections can be easily calculated from the resels of the component tetrahedra, triangles and edges alone.

Results. As a test, the method was applied to detecting local shape differences between the cortex of normal males ($n = 83$) and females ($n = 68$) using smoothed 3D binary masks[3], 2D surface normal displacements[4] and 3D deformation vectors[5]. In all cases the eFWHM varied considerably from 5mm to 30mm depending on location, so the images were highly non-stationary. Despite this, the P-values for local maxima were not greatly affected, but the P-values for cluster sizes were very sensitive to non-stationarity.

References

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