Rotation Space: Detecting Functional Activation by Searching Over Rotated and Scaled Filters

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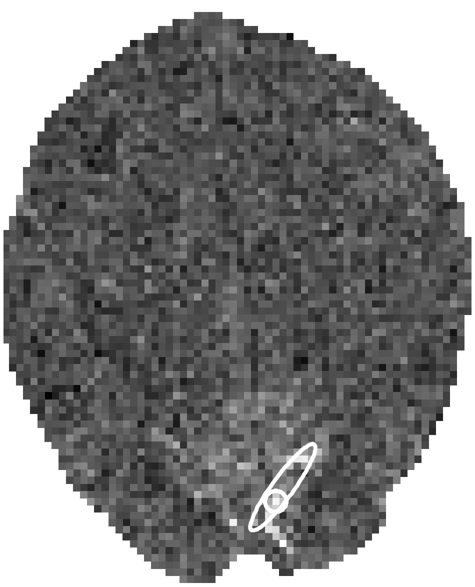
Introduction The aim of this paper is to extend the concept of scale space and multi-resolution analysis to rotation space, that is, rotating as well as scaling a Gaussian-shaped smoothing filter. Using random field theory, we have derived an accurate P-value for the rotation space maximum that allows us to detect non-isotropic signals of arbitrary scale or rotation in functional PET and fMRI images. The advantage of our method is increased sensitivity at detecting elliptically shaped regions of activation that might be missed by a circular shaped filter. These results are applied to a simple fMRI experiment.

Methods In the statistical analysis of functional activation data, the D-dimensional images are often spatially smoothed before analysis by convolution with a Gaussian-shaped filter $f(\mathbf{t})$ to enhance the signal to noise ratio, where $\mathbf{t}$ is a location vector in D dimensional space. The motivation for this comes from the Matched Filter Theorem of signal processing, which states that signal added to white noise is best detected by smoothing with a filter whose shape matches that of the signal. Thus to best detect a 20mm wide signal, a 20mm wide filter should be used. The problem is that the extent of the signal is usually unknown. In practice, PET and fMRI data are often smoothed with a 10-20mm FWHM spherically symmetric Gaussian filter as a first guess.

It is natural to consider searching over filter width as well as location, that is, to use a filter $w^{-D/2}f(\mathbf{t}/w)$ with width w varying over a predetermined interval (w_1, w_2). This adds an extra dimension to the search space, called scale space[1,2]. Theoretical results for the P-value of the scale space maximum[3] can be used to separate signal from noise with a pre-determined false positive rate.

The aim of this paper is to extend this to rotating filters of the form $|\mathbf{W}|^{-D/2}f(\mathbf{W}^{-1}\mathbf{t})$, where $\mathbf{W}$ is now a $D \times D$ lower triangular matrix that rotates and scales the axes of the Gaussian filter. This adds an extra $D(D+1)/2$ dimensions to the search space. The motivation for this is that if the signal in the images has an ellipsoidal Gaussian shape of unknown rotation and axis lengths, then the rotation space maximum should be the most sensitive at detecting it. Our main theoretical result is an accurate P-value for rotation space local maxima for 2D ($D = 2$) data, taking into account multiple testing over voxels, rotation, and scaling.



Results The methods were validated on a simple fMRI experiment in which 2D images were acquired while a subject was presented with a simple on-off visual stimulus. This was cross-correlated with a sine wave with a lag of 8 seconds and a frequency that matched that of the signal. The resulting image was standardised by dividing by a voxel standard deviation based on similar images at frequencies that bracketed that of the stimulus. This approximate z statistic image (Figure; white=high Z) was filtered in rotation space from 6mm to 30mm and the resulting 5D space (2D for location, 1D for rotation and 2D for axis scaling) was searched for local maxima.

The $P = 0.05$ critical threshold for the rotation space Z-statistic image was 5.01, based on the above theory. Several 5D local maxima appeared above this threshold, all in the visual cortex. The 5D global maximum was 16.26 ($P < 0.0001$); the global maximum filter shape was an elongated elliptical filter with axis lengths 6 and 30mm FWHM inclined at an angle of 54° to the x axis. This picked out a 6×30mm elliptically shaped region of activation in the visual cortex (Figure). In contrast, the 3D scale space global maximum Z, searching over circular filters from 6 to 30mm FWHM, was 13.94. This picked out a small circular 6mm diameter region of activation inside that detected by rotation space (Figure).

Conclusions The rotation space critical threshold (5.01) is slightly higher than either the scale space threshold (4.81) from [3] or the usual fixed filter theshold (4.17) from [4]. However this is offset by the greater sensitivity at detecting signals of all rotated elliptical shapes.

References

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