(1) **A Chapter of Scharlau-Opolka**  
You may pick one of the chapters (between ch. 2 and ch. 8) in the book by Scharlau and Opolka and write a project on the mathematician that is featured in that chapter, giving more details to flesh out the mathematics alluded to in the text.

(2) **The Miller-Rabin primality test**  
Explain the Miller-Rabin primality test and Miller’s analysis of its complexity, based on the (generalized) Riemann hypothesis.  
Carl Pomerance, Recent developments in primality testing, Mathematical Intelligencer 3, no. 3 (1981) 97-105.

(3) **The Agrawal-Kayal-Saxena primality test.**  
A few years ago Manindra Agrawal, Neeraj Kayal and Nitin Saxena achieved a major breakthrough by showing that a number N can be tested for primality in time proportional to a polynomial in \( \log(N) \).  
Manindra Agrawal, Neeraj Kayal, Nitin Saxena, PRIMES is in P. Annals of Mathematics 160(2): 781-793, 2004. (Or, you can download the original paper directly.)

(4) **Lenstra’s factorization algorithm based on elliptic curves over finite fields.**  

(5) **Elliptic curve cryptosystems.**  

(6) **Shor’s factorization algorithm based on ”quantum computers”.**  
Recently a probabilistic algorithm was discovered for factoring integers in polynomial time on a quantum computer, a computing device which uses elementary particles to store information and exploits the peculiar properties of these particles (as described by quantum mechanics) to achieve a fast factorization algorithm. This discovery has caused a lot of excitement among number theorists, theoretical computer scientists, and physicists, and represents a remarkable confluence of the three subjects. This would be a good topic for someone who already has some background in physics (and the rudiments of quantum mechanics in particular).  

(7) **The prime number theorem.**  
(8) **Continued fractions and ergodic theory.**
Khintchine, A. Ya. Continued fractions. Translated by Peter Wynn. P. Noordhoff, Ltd.,

(9) **Euler’s calculation of \( \zeta(2k) \) and its developments.**

(10) **The irrationality of \( \zeta(3) \).**

(11) **The theory of partitions.**

(12) **Schoof’s algorithm for counting the number of points on an elliptic curve over a finite field.**

(13) **Computing the zeroes of the Riemann zeta-function.**

(14) **Cubic and biquadratic reciprocity.**

(15) **The Hasse principle, Hasse-Minkowski theorem, and the Brauer-Manin Obstruction for quaternion algebras.**
Bright, Martin. The Brauer-Manin Obstruction, online notes. (In particular, the example by Birch and Swinnerton-Dyer).
(16) **The Lucas-Lehmer test.**

(17) **Geometry of numbers.**

(18) **Transcendental number theory.**

(19) **Brun’s theorem on the upper bound of twin primes.**
Denis Xavier Charles, Sieve Methods, online notes.

(20) **Mahler measures.**
*Start with the following introductory notes, and pick an interesting result, fleshing out the details.*
Lalin, Matilde. Introduction to Mahler measure. online notes.

(21) **Tropical geometry.**
*You will want to restrict your study to objects defined by a single equation, called hypersurfaces.*
Manfred Einsiedler, Mikhail Kapranov, Douglas Lind. Non-archimedean amoebas and tropical varieties.