No calculators are allowed. You may reference and use any results that were stated in class, unless you are explicitly asked to prove it. When you do use them, please state clearly which result you are using.

For Math 346 students, the problem with the lowest score will be dropped. The bonus problem will count as extra credit for everyone.

(1) (3+5+2 points)
   (a) Define what it means for an element \( g \in \mathbb{Z}/n\mathbb{Z} \) to be a primitive root, where \( n \geq 2 \).
   (b) Show that 2 is a primitive root modulo 29.
   (c) What is the structure of \( U(\mathbb{Z}/29\mathbb{Z}) \)?

(2) (6+4 points)
   (a) State and prove Wilson’s theorem.
   (b) On the other hand, show that when \( n \) is not a prime, \((n-1)! \equiv 0 \mod n\), except when \( n = 4 \).

(3) (10 points) Show that
\[
ab(a^2 - b^2)(a^2 + b^2) \equiv 0 \mod 30
\]
for all \( a, b \in \mathbb{Z} \).

(4) (10 points) Let \( u, v \in \mathbb{Z} \), satisfy \( u, v \neq 0 \), and \( u \neq \pm v \). Suppose that \( \gcd(u, v) = 1 \). Show that \( \gcd(u + v, u - v) \) is either 1 or 2.

(5) (10 points) Let \( p \) be a prime. Prove that if \( \text{ord}_p(a) = 3 \), then \( \text{ord}_p(a + 1) = 6 \).

Bonus. (3 points) Prove that \( \sqrt{3} + \sqrt{7} \) is irrational.