(1) Find all integral solutions to $165x^2 - 21y^2 = 19$.

(2) Find all integral solutions to $x^2 - 6y^2 = 1$ and $x^2 - 624y^2 = 1$.

(3) Show that if $x^2 - dy^2 = -1$ has an integral solution, then so does $x^2 - dy^2 = 1$.

(4) Suppose that if $x^2 - dy^2 = n$ with $d > 0$ squarefree has an integral solution $xy \neq 0$. Then show that it has infinitely many solutions.

(5) Let $a + b\sqrt{p}$ be the fundamental solution to $x^2 - py^2 = 1$, where $p$ is a prime congruent to 1 mod 4. Show that $x^2 - py^2 = -1$ has an integral solution using the following steps:
   (a) Show that $a$ is odd.
   (b) Show that $a + 1 = 2u^2$, $a - 1 = 2pv^2$, $2uv = b$.
   (c) Show that $u^2 - pv^2 = \pm 1$.
   (d) Show that in part (c), the negative sign holds.