MATH 579 - ASSIGNMENT 4

Posted	Sunday March 31^{st} 2012
Due	Wednesday April 12^{th} 2012

- *Write the following assignment using LaTeX or the (very easy to use) LaTeX editor: LyX available for free at http://www.lyx.org/.*
- Clarity of the figures and conciseness of explanations will be part of the grade.
- Download the supporting codes on the course webpage.

1. Level Set Method

you can download the code for the initial conditions of both problems: *ic_phi.m*.

1.1. Interface propagation.

in the domain $[0,1]^2$, let

$$\phi_0(x,y)=\sqrt{\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2}-\frac{1}{8}.$$

The zero-contour of ϕ corresponds to a circle of radius $\frac{1}{8}$, centered at $\left(\frac{1}{2},\frac{1}{2}\right)$. Solve the level set equation (using the method presented in class)

$$\phi_t+F\left|\nabla\phi\right|=0$$

from $t=0$ to $t=1$ with initial condition $\phi(x,y,0)=\phi_0(x,y)$, and $F=\sin(2\pi t)$. Present a convergence plot in the L_∞norm and a plot of the zero contour of the solution at $t=1$ on a 100×100 grid.

1.2. Mean curvature flow.

in the domain $[0,1]^2$, let

$$\phi_0(x,y)=\sqrt{\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2}-\left(\frac{1}{4}+\frac{1}{6}\sin\left(\arctan\left(\frac{x-\frac{1}{2}}{y-\frac{1}{2}}\right)\right)\right).$$

Solve the level set equation (using the method presented in class)

$$\phi_t+F\left|\nabla\phi\right|=0$$

from $t=0$ to $t=1$ with initial condition $\phi(x,y,0)=\phi_0(x,y)$, and $F=-\kappa$, where $\kappa=\nabla\cdot\left(\frac{\nabla\phi}{|\nabla\phi|}\right)$ is the mean curvature (note that $\kappa=\kappa(x,y,t)$). Plot the zero contour of the solution at $t=1$ on a 100×100 grid.

At $t=1$ switch the sign of F, i.e. now $F=+\kappa$, and evolve the solution until $t=2$. Plot the zero contour of the solution at $t=2$ along with the initial conditions on a 100×100 grid. Described and explain your observations. Is the system hyperbolic, explain? If it is not hyperbolic, then how would you classify it?

Note: pay special attention to the computation of κ. Recall that in the discrete setting, $|\kappa|<1/h$, where h is the mesh size (e.g. a circle of radius smaller than h cannot be represented by a level set function as defined above on a grid with mesh size h).

2. Navier-Stokes equations

Solve *one* of the following two problems. You may solve both for extra credits.

2.1. **Finite-Difference approach - lid-driven cavity problem.**

Download the code: *mit18086_navierstokes.m*.

Devise a way to determine the global accuracy of the method. For instance, one way to achieve this may be to compute the solution at some fixed time using a *very fine* grid and declare that the exact solution. Explain your method clearly, and produce two convergence plots, one in the L_2 norm and one in the L_∞ norm. Explain your results.

Note: there are a lot of choices to be made here. e.g. T_{final}, finest resolution to be used, sequence of resolutions for the convergence,... Be clear and concise about these choices.

2.2. **Pseudo-spectral approach - homogeneous flow.**

Download the code: *mit18336_spectral_ns2d.m*.

Devise a way to determine the global accuracy of the method. For instance, one way to achieve this may be to compute the solution at some fixed time using a *very fine* grid and declare that the exact solution. Explain your method clearly, and produce two convergence plots, one in the L_2 norm and one in the L_∞ norm. Explain your results.

Note: there are a lot of choices to be made here. e.g. T_{final}, finest resolution to be used, sequence of resolutions for the convergence,... Be clear and concise about these choices.