

**MCGILL UNIVERSITY
FACULTY OF SCIENCE**

PRACTICE MIDTERM EXAM

MATH 150

Examiner: Professor V. Jaksic

Time: 120 min

1. Fill in the blank with the correct result and state the requested test.

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- In the space below state the **Second Derivative Test**

- The absolute minimum of the function $f(x) = x^x$ on $[0, \infty)$ is

_____.

- The point on the line $y = 4x + 7$ that is closest to the origin is

_____.

2. Fill in the blank with the correct result and state the requested theorem.

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- In the space below state **L'Hospital's Rule**

- Let $f(x) = (x - 3)^{-2}$. Is there a number c in $(1, 4)$ such that $f(4) - f(1) = 3f'(c)$? Write a short justification of your answer.

- Find the intervals of concavity of the function

$$f(x) = e^{-\frac{1}{x+1}}$$

Concave upward on _____ Concave downward on _____

3. Fill in the blank with the correct result and state the requested theorem.

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- In the space below state **The Mean Value Theorem**

- The number of solutions of the equation $3x + 2 \cos x + 5 = 0$ is

- The value of the limit

$$\lim_{x \rightarrow \frac{\pi}{2}^-} (\tan x)^{\cos x}$$

is

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4. Show that the length of the portion of any tangent line to astroid $x^{2/3} + y^{2/3} = a^{2/3}$ cut off by the coordinate axis is constant. Justify carefully your answer.

5. Show that for all $x > 0$,

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$$\frac{x}{1+x^2} < \arctan x < x$$

Justify carefully your answer.

6. For what values of a, b, c, d the function

$\bar{0}$ $f(x) = ax^3 + bx^2 + cx + d$
has a local maximum value of 3 at -2 and a local minimum value of 0 at 1? Justify your answer.

7. Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$. Justify carefully your answer.

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