

**MCGILL UNIVERSITY
FACULTY OF SCIENCE**

MIDTERM EXAM

MATH 150

Examiner: Professor V. Jaksic

Time: 120 min

Family Name (Please Print): _____

First Name: _____

Student Number: _____ Tutorial Section: _____ ■

No notes or books are allowed.

Calculators are not permitted.

If you need extra space, use the back of the paper for rough work.

There are six empty pages at the end of the exam.
You may use them for rough work. These pages will not be graded.

There are 7 questions worth a total of 200 points.

Partial marks will be given for all questions

Write your solutions in a clear, complete and logical way.

1. Fill in the blank with the correct result and state the requested result.

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- [10 points] In the space below state the **Concavity Test**

- [10 points] Find the points on the ellipse $2x^2 + y^2 = 1$ where the tangent line has slope 1.

The points are _____.

- [10 points] The value of the limit

$$\lim_{x \rightarrow \frac{\pi}{2}^-} \left(x - \frac{\pi}{2}\right) \tan x$$

is

2. Fill in the blank with the correct result and state the requested theorem.

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- [10 points] In the space below state **Fermat's Theorem**

- [10 points] Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x^2 + 1$$

on the interval $[-\frac{1}{2}, 4]$.

The absolute maximum is _____ The absolute minimum is _____

- [10 points] Find the intervals of concavity of the function

$$f(x) = e^{\arctan x}$$

Concave upward on _____ Concave downward on _____

3.

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- [10 points] On the line below complete the statement of the theorem: **If $f'(x) = 0$ for all x in an interval (a, b) , then**

- [10 points] The number of solutions of the equation $3x + x^4 + x^6 = 0$ is

- [10 points] The value of the limit

$$\lim_{x \rightarrow 0^+} (\sin x)^x$$

is

4. For what values of a and b is the following equation true?

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$$\lim_{x \rightarrow 0} \left(\frac{\sin 2x}{x^3} + a + \frac{b}{x^2} \right) = 0.$$

Justify carefully your answer.

5. Show that for all $x \geq 0$,

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$$\arcsin \frac{x-1}{x+1} = 2 \arctan \sqrt{x} - \frac{\pi}{2}.$$

Justify carefully your answer.

6. Show that for all $x > 0$,

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$$\sqrt{1+x} < 1 + \frac{1}{2}x.$$

Justify carefully your answer.

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7. Find the dimensions of the rectangle of the largest area that has its base on the x -axis and its other two vertices above the x -axis are lying on the curve

$$y = 2009 - x^{2010}.$$

Justify carefully your answer.

Rough work.

Rough work.

Rough work.

Rough work.

Rough work.

Rough work.