

PROBLEM SET 3

The problems are taken from Chapter 2 of the Lecture Notes by W.W.L. Chen, located at

<http://www.maths.mq.edu.au/~wchen/lnentfolder/ent02-af.pdf>

Problem 1. Chen, chapter 2, # 4. Prove that

$$\prod_{m|n} m = n^{d(n)/2}.$$

Problem 2. Chen, chapter 2, # 13. Prove that

$$\sum_{m|n} \mu(m) \sigma\left(\frac{n}{m}\right) = n$$

for every $n \in \mathbf{N}$.

CONVOLUTIONS. This material is discussed at the end of Chapter 2 in W.W.L. Chen's notes, §2.5.

Given arithmetic functions $f, g, h : \mathbf{N} \rightarrow \mathbf{C}$, define the *convolution* operation $f * g$ as follows:

$$(f * g)(n) := \sum_{d|n} f(d)g\left(\frac{n}{d}\right).$$

Problem 3. Prove that

- a) $f * g = g * f$ (* is commutative);
- b) $(f * g) * h = f * (g * h)$ (* is associative).

Problem 4. Show that if f, g are multiplicative, then so is $f * g$.

Next, define $\mathbf{1}(n) = 1$ for all n ; and

$$\mathbf{I}(n) = \begin{cases} 1, & \text{if } n = 1, \\ 0, & \text{if } n \neq 1. \end{cases}$$

Then

$$(f * \mathbf{1})(n) = \sum_{d|n} f(d).$$

Also, Möbius inversion formula is equivalent to $\mathbf{1} * \mu = \mathbf{I}$.