Math 565, Winter 2009 D. Jakobson

REVIEW TOPICS (MATH 564-5 AND QUALS)

- Math 564 material: Lebesgue integration; dominated/monotone convergence; Fatou's lemma; Lusin and Vitali-Caratheodory theorems; Jensen's, Holder and Minkowski inequalities; inner/outer regularity of measures; L^p spaces and their duals; topology: complete, (locally) compact, continuous, separable; F_σ and G_δ sets; upper/lower semicontinuous; σ-finite; convexity; Riesz representation theorem; Baire's theorem; Hilbert spaces; Fourier transform on $\mathbf{T}$, Parseval, Riesz-Fischer; Riemann-Lebesgue lemma, Dirichlet and Fejer kernels, elementary convergence results for Fourier series; linear functions; Hahn-Banach, Banach-Steinhaus and Open Mapping theorems + elementary applications (e.g. range of Fourier transform from L^1).
- Complex measures: total variation, positive/negative variation, Jordan decomposition.
- Absolute continuity of measures, Lebesgue-Radon-Nikodym and Hahn decomposition theorems.
- Derivatives of measures, covering lemma, maximal function and L^1 maximal inequality.
- Lebesgue points, metric density, Lebesgue differentiation theorem; differentiation of complex Borel measures.
- Fundamental theorem of calculus, absolute continuity, functions of bounded variation, change of variables.
- Product σ-algebras and measures, Fubini theorem, completion of product measures.
- Convolutions, distribution function, L^p maximal estimate, $p > 1$; L^p properties of convolutions.
- Fourier transform in $\mathbf{R}$, L^1 and L^2 theory; Fourier inversion; elementary Fourier transforms; Plancherel theorem; translation-invariant subspaces of L^2; Banach algebra L^1, complex homomorphisms on L^1.
- Poisson summation formula.
- Chapter 11 (harmonic functions), not required for quals but covered in the course: Cauchy-Riemann equations, holomorphic functions; harmonic functions; Poisson integral and Poisson kernel; Harnack inequality and

Harnack theorem; mean value property; Schwarz reflection principle; radial and nontangential limits and associated maximal functions, maximal estimates; boundary behavior of Poisson integrals, representation theorems; Arzela-Ascoli theorem, uniform equicontinuity, application (e.g. to functionals on a separable Banach space, or to the embedding $C^k \to C^{k-1}$).