

PROBLEM SET 1

due Wednesday, September 17, by 5pm

Do all the problems. Every problem is worth 5 points. Some problems will not be graded because of time constraints.

Rudin, chapter 1, # 2, 6, 9, 10, 12.

Problem 6 Let $f : X \rightarrow \mathbf{R}$ be a function. Describe all those $n \in \mathbf{N}$ for which measurability of $f(x)^n$ implies measurability of $f(x)$.

Problem 7 Let f be a continuous function on $[0, 1]$. Prove that the function $g(c) = \#\{x \in [0, 1] : f(x) = c\}$ is Lebesgue measurable.

Problem 8 (extra credit). Give an example of a continuous $f : \mathbf{R} \rightarrow \mathbf{R}$ and measurable $g : \mathbf{R} \rightarrow \mathbf{R}$ such that $g(f(x))$ is not measurable.