189-265A: Advanced Calculus

OLD MIDTERM EXAMS AND CLASS TESTS

Midterm Exam October 1996

Answer all questions in Part A. Answer two (2) question from Part B and one (1) question from Part C for a total of six (6) questions.

PART A: Answer each of the following three (3) questions

A1 Compute

(a) $\int_C (x+y)dx - (x-y)dy + (x+y+z)dz$ if C is parametrized by $x = t, y = t^2, z = -t$, where t goes from -1 to 1.

(b) $\int_C \{y^2 - y\sin(xy)\}dx + \{2xy - x\sin(xy)\}dy$, where C is the line segment from $(3,0)$ to $(0,3)$.

A2 Calculate the line integral $\int_C (-\frac{1}{3}y^3 - x^5)dx + (\frac{1}{3}x^3 + y^4)dy$, where C denotes the unit circle $x^2 + y^2 = 1$ traversed once counterclockwise.

A3 Compute $\int\int_D (x-y)dxdy$, where D is the region bounded by $2y - x = 3, 2y - x = 1, x + y = 5$, and $x + y = 4$.

PART B: Answer two (2) of the following questions

B1 A wire along the 1st quadrant part of $x^2 + y^2 = a^2$ has density $\delta(x,y)$ given by $\delta(x,y) = xy$. Find the position of the center of mass.

B2 Compute $\int_C \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$, where C is

(a) the circle $x^2 + y^2 = a^2$ traversed once counterclockwise, and

(b) the perimeter of the triangle with vertices $(1,0), (-1,1), (0,-1)$. Justify your calculation.

B3 Use a line integral to compute the area of the triangle with vertices at $(0,0), (2,-3)$, and $(3,4)$. Then calculate the area of the same triangle using a double integral (or integrals).

B4 If $\overrightarrow{F}(x,y) = (y+x, x)$ compute the flux of $\overrightarrow{F}$

(a) across the line segment from $(1,1)$ to $(3,-1)$ (observe the standard orientation convention of the assignments), and

(b) across the boundary of the triangle with vertices at $(1,0), (0,1)$, and $(0,0)$ (in the outward sense).

PART C: Answer one (1) of the following questions

C1 Find the maximum value of $xy + yz$ if $x^2 + y^2 + z^2 = 1$.

C2 If $f(x, y, u, v) = x - e^u \cos v$ and $g(x, y, u, v) = y - e^u \sin v$ show that near $x = \frac{1}{\sqrt{2}}, y = \frac{1}{\sqrt{2}}, u = 0, v = \frac{\pi}{4}$, one can solve the equations

$$f(x, y, u, v) = 0$$
$$g(x, y, u, v) = 0$$

for x and u as functions of y and v. Determine

$$\begin{bmatrix} \frac{\partial x}{\partial y} & \frac{\partial x}{\partial u} \\ \frac{\partial v}{\partial y} & \frac{\partial v}{\partial u} \end{bmatrix}$$

for $x = \frac{1}{\sqrt{2}}, y = \frac{1}{\sqrt{2}}, u = 0, v = \frac{\pi}{4}$.

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Midterm Exam October 1997

Answer all questions in Part A. Answer two (2) question from Part B and one (1) question from Part C for a total of six (6) questions.

PART A: Answer each of the following three (3) questions

A1 Compute

(a) $\int_C (x + z)dx - (x - z)dy - (x + y - z)dz$ if C is parametrized by $x = t, y = t, z = t^2$, where t goes from 0 to 1.

(b) $\int_C \{xy^2 - ye^{xy}\}dx + \{x^2y - xe^{xy}\}dy$, where C is the line segment from $(0, 0)$ to $(1, 0)$ followed by the arc of the circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(0, 1)$ that lies in the first quadrant.

A2 Calculate the line integral

$$\int_C (-\frac{1}{2}y^2 - x^5)dx + (\frac{1}{2}x^2 + y^4)dy,$$

where C denotes the boundary of the rectangle determined by $(0, 0), (4, 0), (4, 2)$, and $(0, 2)$ traversed once **clockwise**.

A3 Compute $\int\int_D (x - y)dxdy$, where D is the triangle with vertices at $(1, 1)$, $(3, -2)$ and $(4, 6)$.

Part B: Answer any two (2) of the following questions.

B1 A wire in the shape of a coil C of a helix is parametrized by $\vec{x}(t) = (\cos t, \sin t, 2t),\ 0 \le t \le 2\pi$. Assume the mass density of the wire is $\delta(x, y, z) = z + 1$. Find the z-coordinate of the center of mass of the wire.

B2 Compute $\int_C \frac{-y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy$, where C is

(a) the circle $x^2 + y^2 = 4$ traversed once counterclockwise, and

(b) the perimeter of the square determined by $(3, 3), (-3, 3), (-3, -3)$, and $(3, -3)$. Justify your calculation.

B3 Use a line integral to compute the area of the triangle with vertices at $(0, 0), (3, -4)$, and $(6, 2)$. Then calculate the area of the same triangle using a double integral (or integrals).

B4 If $\overrightarrow{F}(x, y) = (x^2 + y^2, xy)$ compute the flux of $\overrightarrow{F}$

(a) across the line segment from $(1, 1)$ to $(3, -1)$ (observe the standard orientation convention of the assignments), and

(b) across the boundary of the triangle with vertices at $(1, 0)$, $(0, 4)$, and $(0, 0)$ (in the outward sense).

PART C: Answer one (1) of the following questions

C1 Find the maximum value of xyz if $x^2 + y^2 + z^2 = 9$.

C2 If $f(x, y, u, v) = x^2 - yu + v^2$ and $g(x, y, u, v) = y^2 + xv - u^2$ show that near $x = 1, y = -1, u = 0, v = 1$, one can solve the equations

$$f(x, y, u, v) = 2$$
$$g(x, y, u, v) = 2$$

for x and u as functions of y and v. Determine

$$\begin{bmatrix} \frac{\partial x}{\partial y} & \frac{\partial x}{\partial v} \\ \frac{\partial u}{\partial y} & \frac{\partial u}{\partial v} \end{bmatrix}$$

when $x = 1, y = -1, u = 0, v = 1$.

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265 Midterm class test November 1998

Instructions: Answer all questions. Note that in question 4 there is a choice.

1. Compute the following line integrals:

(a) $\int_C x^2 y dx + xy dy$, where C is the line segment from $(2, 0)$ to $(6, 5)$.

(b) $\int_C (y+z+yz\cos(xyz))dx + (x+z+xz\cos(xyz))dy + (x+y+xy\cos(xyz))dz$, where C is the line segment from $(0,2,0)$ to $(1,1,1)$ followed by the line segment from $(1,1,1)$ to $(3,6,1)$ followed by the line segment from $(3,6,1)$ to $(1,\pi/2,1)$.

2. Calculate $\int_C (y^2-x^2)dx+3xydy$, where C is the curve $(x-1)^2+y^2=1$ traversed once counterclockwise.

3. Compute the outward flux of $\overrightarrow{F}(x,y)=(xy,-xy)$ across the boundary of the triangle with vertices at $(1,-1),(5,-1)$, and $(3,1)$.

4. **Either** compute the line integral $\int_C \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$ where C is

   (a) the boundary of the square with corners at $(1,-1),(3,-1),(3,1)$ and $(1,1)$ traversed counterclockwise.

   (b) the boundary of the square with corners at $(-1,-1),(1,-1),(1,1)$ and $(-1,1)$ traversed counterclockwise.

   (c) the boundary of the rectangle with corners at $(-1,-1),(3,-1),(3,1)$, and $(-1,1)$.

   (d) the line segment from $(0,1)$ to $(-1,1)$.

   Explain your calculations.

   Or compute the outward flux of $\overrightarrow{F}(x,y)=(\frac{x}{x^2+y^2},\frac{y}{x^2+y^2})$ across

   (a) the boundary of the triangle T with vertices at $(-2,2),(2,2)$ and $(0,4)$.

   (b) the boundary of the square S with vertices at $(2,2),(-2,2),(-2,-2)$ and $(-2,2)$.

   (c) the boundary of the union of the square S and the triangle T.

   (d) the line segment from $(-2,2)$ to $(2,2)$ using the convention that the normal is to the right of the curve.

   Explain your calculations.

5. Find the maximum value of $3x-5y+z$ on the sphere $(x+2)^2+(y-1)^2+(z+4)^2=4$.

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Midterm Class test October 1999

1. Compute the line integral

$$\int_C \left(\frac{-y}{x^2+y^2} - y\sin(xy) + y^4\right)dx + \left(\frac{x}{x^2+y^2} - x\sin(xy)\right)dy$$

   where C is the boundary of the triangle with vertices at $(1,-1),(-1,2)$ and $(-1,-1)$, taken counterclockwise.

2. Use Green's formula to compute the area enclosed by the curve

$$x(t)=(2+\cos t)\cos t, \quad y(t)=(2+\cos t)\sin t, 0\le t\le 2\pi.$$

3. Explain why (u, v) can be solved for as functions of (x, y) near the point $(x_0, y_0, u_0, v_0) = (1, 1, 1, 1, 1)$ from the relations

$$\begin{aligned} xu + yvu^2 &= 2 \\ xu^3 + y^2v^4 &= 2 \end{aligned}$$

Compute $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ at $(1, 1, 1, 1)$.