1. (a) If C is the curve $x = \pi(1 + 2t^4)$, $y = (t + 2)\cos \pi t$, $z = 1 + t^5$, $(0 \le t \le 1)$, compute

$$\int_C (y^2 \cos x + z^3)\, dx + (2y \sin x - 4)\, dy + (3xz^2 + 2)\, dz.$$

(b) Show that $\vec{F} = 2xz\vec{i} + (x^2 - y)\vec{j} + (2z - x^2)\vec{k}$ is not conservative by finding points A, B and curves C_1, C_2 joining A to B such that

$$\int_{C_1} \vec{F} \cdot d\vec{r} \ne \int_{C_2} \vec{F} \cdot d\vec{r}.$$

2. Compute the line integral

$$\int_C \left(\frac{-\sqrt{2}(y-1)}{x^2 + 2(y-1)^2} + y^2 \right) dx + \left(\frac{\sqrt{2}x}{x^2 + 2(y-1)^2} + 2xy \right) dy$$

over

(a) the ellipse $x^2 + 2(y-1)^2 = 1$ with counterclockwise orientation;

(b) the circle $x^2 + (y-1)^2 = 4$ with clockwise orientation. (Hint: Use the result of (a) and Green's Theorem.)

3. Using Stokes Theorem, compute the line integral

$$\int_C (y + 2z)\, dx + (z + 2x)\, dy + (x + 2y)\, dz,$$

where C is the intersection of $z = x^2 + y^2$ and $z = 2x + 2y + 2$ with the counterclockwise orientation (viewing the curve from above).

4. Use the method of Lagrange multipliers to find the distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225$.

5. Compute the area of that portion of the cone $x^2 + y^2 = 3z^2$ which lies inside the cylinder $x^2 + y^2 = 4y$.

6. Compute the flux of the vector field

$$\vec{F} = xy^2\vec{i} + x^2y\vec{j} + y\vec{k} + \nabla\left(\frac{1}{\sqrt{x^2 + y^2 + (z - 1/2)^2}} \right)$$

across the closed surface consisting of the cylinder $x^2 + y^2 = 1$, $-1 \le z \le 1$ and the two "lids" $x^2 + y^2 \le 1$, $z = \pm 1$, oriented with the outward-pointing normal.