Math 264: Advanced Calculus **Winter 2008**

Assignment 4 **due Thursday, March 20**

Every problem is worth 5 points. Due to time constraints, some problems may not be marked.

In Problems 1 and 2, D is a domain in $\mathbf{R}^3$ satisfying the conditions of divergence theorem, S is the boundary of D, and $\mathbf{N}$ is the outward unit normal to S. The functions ϕ and ψ are smooth on $D \cup S$. Finally, for any smooth f, the normal derivative of f, $\partial f/\partial n$, is defined by $\partial f/\partial n = \nabla f \bullet \mathbf{N}$.

Problem 1 (Adams, §16.4 # 23). If $\mathbf{F}$ is a smooth vector field on a domain D, show that

$$\int\int\int_D [\phi(\mathrm{div}\mathbf{F}) + \nabla\phi \bullet \mathbf{F}]\, dV = \int\int_S \phi\mathbf{F} \bullet \mathbf{N}\, dS.$$

Problem 2 (Adams, §16.4 # 28). Recall that Δf denotes the Laplacian of f. Verify that

$$\int\int\int_D [\phi(\Delta\psi) - \psi(\Delta\phi)]\ dV = \int\int_S \left[\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n}\right]\ dS.$$

Problem 3 (Adams, §16.5 # 2). Evaluate $\oint_C ydx - xdy + z^2dz$ around the curve C of the intersection of the cylinders $z = y^2$ and $x^2 + y^2 = 4$, oriented counterclockwise as seen from a point high on the z-axis.

Problem 4 (Adams, §16.5 # 6). Evaluate $\oint_C \mathbf{F} \bullet d\mathbf{r}$ around the curve

$$\mathbf{r}(t) = (\cos t, \sin t, \sin(2t)), \qquad 0 \le t \le 2\pi,$$

where $\mathbf{F} = (e^x - y^3, e^y + x^3, e^z)$. Hint: show that C lies on the surface $z = 2xy$.

Problem 5 (Adams, §16.5 # 10). Let C be the curve $(x-1)^2 + 4y^2 = 16, 2x + y + z = 3$, orienetd counterclockwise when viewed from high on the z-axis. Let

$$\mathbf{F} = (y^2 + z^2 + \sin(x^2), 2xy + z, xz + 2yz).$$

Evaluate $\oint_C \mathbf{F} \bullet d\mathbf{r}$.