

Assignment 2

due Thursday, February 7

Every problem is worth 5 points. Due to time constraints, some problems may not be marked.

Problem 1 (Adams, §14.6 # 21). Compute the volume of the region lying in the first octant, between the planes $y = 0$ and $y = x$, and inside the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. Hint: use the change of variables suggested in Adams, Example 1, section 14.6.

Problem 2 (Adams, §15.2 # 10). Show that the vector field

$$\mathbf{F}(x, y, z) = (2x/z, 2y/z, 1 - (x^2 + y^2)/z^2)$$

is conservative and find its potential. Describe the equipotential surfaces. Find the field lines of \mathbf{F} .

Problem 3 (Adams, §15.3 # 8). Find $\int_C \sqrt{1 + 4x^2z^2} ds$, where C is the curve of intersection of the surfaces $x^2 + z^2 = 1$ and $y = x^2$.

Problem 4 (Adams, §15.4 # 13). If C is the intersection of $z = \ln(1 + x)$ and $y = x$ from $(0, 0, 0)$ to $(1, 1, \ln 2)$, evaluate

$$\int_C (2x \sin(\pi y) - e^z) dx + (\pi x^2 \cos(\pi y) - 3e^z) dy - xe^z dz.$$

Problem 5 (Adams, §15.5 # 15). Find $\int \int_U xz dS$, where U is the part of the surface $z = x^2$ that lies in the first octant of \mathbf{R}^3 and inside the paraboloid $z = 1 - 3x^2 - y^2$.

Problem 6 (Adams, §15.6 # 7). Find the flux of $\mathbf{F} = (y^3, z^2, x)$ through the part of the surface $z = 4 - x^2 - y^2$ that lies above the plane $z = 2x + 1$.