Department of Mathematics and Statistics
McGill University
Math 262 Practice Midterm
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INSTRUCTIONS

- You have TWO HOURS to complete the exam.
- Please show how your answers are derived. A correct solution without work will NOT receive full mark.
- Please read each question carefully and answer all questions neatly in the place provided.
- Non-programmable calculators are permitted.
- Formula sheets are not permitted.
- PLEASE NOTE: Invigilators are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

1. Calculate the following limits.
   (a) $\lim\limits_{n\to\infty} \dfrac{4^n}{n!}$
   (b) $\lim\limits_{n\to\infty} \dfrac{\arctan(n)\cdot(n-1)^n}{n^n}$
   (c) $\lim\limits_{n\to\infty} \dfrac{n^{1/3}\cdot\ln(n^{-2014})}{(n^2+5n+2)^{1/6}\cdot e^{\ln\ln n}}$.

2. (a) Determine if the series $\sum\limits_{n=1}^{\infty} \sqrt{n}\cdot\sin(1/n)$ converges or diverges. Hint: first compute the limit $n\cdot\sin(1/n)$.
   (b) Determine if the series $\sum\limits_{k=3}^{\infty} \dfrac{1}{k(\ln k)^{1/2}}$ converges or diverges.

3. (a) Use the integral test to estimate the difference between the partial sum S_{12} of the series $\sum\limits_{n=1}^{\infty} \frac{1}{n^4}$, and the sum of the series.
   (b) Use the Taylor series of $\cos x$ to estimate the difference between $\cos(\pi/5)$ and the number $1-\pi^2/(2\cdot 25)+\pi^4/(24\cdot 5^4)$.

4. (a) Find all the values of x for which the series $\sum\limits_{n=1}^{\infty} \frac{(x^2-1)^n}{2^n}$ converges.
   (b) Find the sum of the series $\sum_{n=0}^{\infty} x^{n+2}/n!$. Hint: recall the Taylor series of e^x.

5. (a) Find the Taylor series of the function $f(x)=\int_0^x (y^2\cos(y))\,dy$ near the point $x=0$.

6. (a) Consider the space curve
$$\mathbf{r}(t)=\langle e^t, \sqrt{2}t, e^{-t}\rangle$$
   where $0\le t\le 2$. Find $\mathbf{T}$ and the curvature at the point $(1,0,1)$. Find the length of the curve.