McGill University — Math 262: Intermediate Calculus, Fall 2014

WRITTEN ASSIGNMENT 4 — Due December 1, 2014

INSTRUCTIONS: Answer any 5 of the following 6 questions. To get the full mark, it is not enough to state the correct answer; there should be a detailed explanation for that answer. You can use any result from the book or from the lectures, but you should explain how it applies to the problem.

Problem 1. (4 points)

(a) Evaluate the integral by reversing the order of integration:

$$\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1+\cos^2 x}\; dxdy.$$

(b) Compute

$$\int_0^{\pi/2} dy \int_y^{\pi/2} \frac{\sin x}{x} dx.$$

Problem 2. (4 points)

(a) Find the mass and the centre of mass of the lamina that occupies the region D bounded by $y = x^2$ and $y = x + 2$, and the density $\rho(x, y) = kx$.

(b) Find the area of a surface formed by the intersection of the cylinders $y^2 + z^2 = 1$ and $x^2 + z^2 = 1$.

Problem 3. (4 points)

(a) Evaluate the integral by changing to spherical coordinates:

$$\int_{-a}^{a} \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} (x^2z + y^2z + z^3)\; dzdxdy.$$

(b) Evaluate the integral by making an appropriate change of variables:

$$\int\int_R \cos\left(\frac{y-x}{y+x}\right) dA,$$

where R is the trapezoidal region with vertices $(1, 0), (2, 0), (0, 2)$ and $(0, 1)$.

Problem 4. (4 points)

(a) Use polar coordinates to find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

(b) Find the area of the conical surface $3z^2 = x^2 + y^2, 0 \leq z \leq 2$.

Problem 5. (4 points)

(a) Find $\int\int\int_R (x^2 + y^2 + z^2) dV$, where R is the region that lies above the cone $z = c\sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = a^2$.

(b) Compute $\int\int\int_R (x^2 + y^2) dV$, where R is the region in (a).

Problem 6. (4 points)

(a) Find the volume of the region lying inside both the sphere $x^2 + y^2 + z^2 = 2a^2$ and the cylinder $x^2 + y^2 = a^2$.

(b) Let T be the triangle with vertices $(0,0), (1,1)$ and $(1,2)$. Find

$$\int\int_T \frac{dxdy}{x\sqrt{y}}.$$