McGill University Math 262: Intermediate Calculus, Fall 2014

WRITTEN ASSIGNMENT 2 Due Octber 20, 2014

Professor: Dmitry Jakobson

INSTRUCTIONS: Answer any 5 of the following 6 questions. To get the full mark, it is not enough to state the correct answer; there should be a detailed explanation for that answer. You can use any result from the book or from the lectures, but you should explain how it applies to the problem.

Problem 1. (4 points) Consider the space curve $\mathbf{r}(t) = (3t^2, 6t, 3\ln t)$, where $1 \leq t \leq 3$.

a) Find $\mathbf{r}'(t), \mathbf{r}''(t)$; compute the arc-length of $\mathbf{r}(t)$.

b) Find $\mathbf{T}$ and the curvature κ at $t = 1$.

Problem 2. (4 points) Find $\mathbf{T}, \mathbf{N}, \mathbf{B}$ for the following curves:

a) $\mathbf{r}(t) = (t^2, 2t^3/3, t)$ at $t = 1$;

b) $\mathbf{r}(t) = (\cos t, \sin t, \ln \cos t)$ at $t = 0$.

Problem 3. (4 points) Verify the following identities:

a) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$;

b) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \mathbf{v} \cdot (\mathbf{w} \times \mathbf{u}) = \mathbf{w} \cdot (\mathbf{u} \times \mathbf{v})$.

You can use any properties of the determinant that you know.

Problem 4. (4 points)

a) Find the distance between the lines $x + 2y = 3, y + 2z = 3$ and $x + y + z = 6, x - 2z = -5$.

b) Show that the line $x - 2 = (y + 3)/2 = (z - 1)/4$ is parallel to the plane $2y - z = 1$. What is the distance between the line and the plane?

Problem 5. (4 points)

a) Express the length of the curve $\mathbf{r} = (at^2, bt, c \cdot \ln t), 1 \leq t \leq T$ as a definite integral. Evaluate the integral if $b^2 = 4ac$.

b) Find the arc length parametrization of the curve $\mathbf{r} = (3t\cos t, 3t \sin t, 2\sqrt{2}t^{3/2})$.

Problem 6. (4 points)

Find $\mathbf{T}, \mathbf{N}, \mathbf{B}$, curvature and torsion at a general point on the curve $\mathbf{r} = (e^t \cos t, e^t \sin t, e^t)$.