McGill University Math 262: Intermediate Calculus, Fall 2014

WRITTEN ASSIGNMENT 1 Due September 29, 2014

Professor: Dmitry Jakobson

**INSTRUCTIONS:** Answer any 5 of the following 6 questions. To get the full mark, it is not enough to state the correct answer; there should be a detailed explanation for that answer. You can use any result from the book or from the lectures, but you should explain how it applies to the problem.

**Problem 1. (4 points)**

Find the limit of the following sequences.

(a)

$$a_n = \left(\frac{n-3}{n}\right)^n$$

(b)

$$a_n = (n^2 + 2n + 3) \cdot \sin\left(\frac{1}{4n^2 + 5n + 3}\right).$$

**Problem 2. (4 points)**

Determine whether the following series converges or diverges by using any appropriate test.

(a)

$$\sum_{n=1}^{\infty} \frac{n^n}{\pi^n \cdot n!}$$

(b)

$$\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}.$$

**Problem 3. (4 points)**

Determine whether the following series converges or diverges.

(a)

$$\sum_{n=1}^{\infty} \frac{2^{2n}(n!)^2}{(2n)!}$$

Hint: compare $a_n$ with $1/(2n)$.

(b)

$$\sum_{n=27}^{\infty} \frac{1}{n \cdot \ln n (\ln \ln n)(\ln \ln \ln n)^{1.5}}$$

**Problem 4. (4 points)** *Fibonacci numbers* $f_n$ are defined as follows: $f_1 = 1, f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$. Consider the power series

$$F(x) = \sum_{n=1}^{\infty} f_n x^n.$$

Show that

$$F(x) = \frac{x}{1 - x - x^2}.$$

Hint: Multiply $F(x)$ by $(1 - x - x^2)$ and use the recursion relations.

**Problem 5. (4 points)** Let $F(x) = \int_0^x \sin(t^2) dt$.

- Find the Maclaurin series for $F(x)$.
- Approximate $F(0.1)$ with an error smaller than 0.001.

**Problem 6. (4 points)**

- Find

$$\lim_{x \to 0} \frac{\sin(\sin x) - x}{x(\cos(\sin x) - 1)}.$$

- Find MacLaurin series representation for the function

$$(4 + x^4)^{-1/3}.$$