How fast and in what sense(s) does
the Calderón reproducing formula converge?

Abstract.

Let $\psi \in \mathcal{C}_0^\infty(\mathbf{R}^d)$ be real, radial, with support contained in $\{x : |x| \leq 1\}$, have integral equal to 0, and satisfy

$$\int_0^\infty |\hat{\psi}(y\xi)|^2 \, \frac{dy}{y} = 1$$

for all $\xi \neq 0$. The "standard" *Calderón reproducing formula* (CRF) is, essentially, the assertion that, for all "reasonable" f,

$$f(x) = \int_{\mathbf{R}_+^{d+1}} (f * \psi_y(t)) \, \psi_y(x-t) \, \frac{dt\, dy}{y}. \tag{1}$$

Formula (1) is a standard tool in harmonic analysis. Unfortunately, the senses in which (1) is true, how the integral converges, and what constitutes a "reasonable" f are frequently left a little vague.

In this talk we will show that a very general form of the CRF integral converges in $L^p(w)$, for all $1 < p < \infty$, whenever w belongs to the Muckenhoupt class A_p. We show that the integral converges whether we interpret it, in very natural senses, as a limit of $L^p(w)$-valued Riemann or Lebesgue integrals. We give quantitative estimates on their rates of convergence (or, equivalently, on the speed at which the errors go to 0) in $L^p(w)$.