A Probabilist's Thoughts about a Theorem of L. Hörmander

A key result in modern theory of elliptic equations is Weyl's Lemma, which says that if u is a distribution and $(\partial_t - \Delta)u = 0$ in some open region, then u smooth in that region. By the 1960's, Weyl's seminal result had been generalized to cover the heat equation associated with any strictly elliptic operator having smooth coefficients. However, until Hörmander's 1967 paper on subelliptic operators there was no theory that covered a famous example found by A.N. Kolmogorov of a hideously non-elliptic operator for which a Weyl type result holds. In this lecture, I will not attempt to prove Hörmander's theorem. Instead, I will give a probabilistic explanation for Kolmogorov's and related examples.