

Valuations on Convex Sets and Sobolev Functions

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Since Dehn's solutions of Hilbert's Third Problem in 1902, valuations on convex sets have played a prominent role in geometry. Here a function z defined on a family \mathcal{C} of convex sets and taking values in an abelian semigroup is called additive, if

$$z(K) + z(L) = z(K \cup L) + z(K \cap L)$$

for all $K, L \in \mathcal{C}$ whenever $K \cup L, K \cap L \in \mathcal{C}$. The most celebrated result is Hadwiger's classification of continuous and rigid motion invariant additive functionals. In the talk, an overview of results on additive functions will be given. Recent results on convex-body valued additive functions will be described and applications of these results in establishing new Sobolev inequalities will be discussed.